THE "DROP OF HONEY EFFECT" METAPHOR IN CHAOS THEORY

Manuel Alberto M. Ferreira, José António Filipe

Instituto Universitário de Lisboa (ISCTE-IUL), Business Research Unit (BRU-IUL) and Information Sciences, Technologies and Architecture Research Center (ISTAR-IUL), ***PORTUGAL***

E-mails: manuel.ferreira@iscte.pt, jose.filipe@iscte.pt

ABSTRACT

Chaos theory resulted in great part from natural scientists' outcomes in non-linear dynamics area. The prominence of associated models has increased in the last decades, due the non-linear systems temporal evolution study. Consequently, chaos is one of the concepts that most quickly have been expanded in what research topics concerns. In case of unstable relationships in non-linear systems, chaos theory aims to understand and explain this kind of unpredictable aspects of nature, social life, the uncertainties, the nonlinearities, the disorders and confusions. Scientifically it represents a disarray connection, but basically it involves much more. The close association between change and time seems essential to understand what happens in the fundamentals of chaos theory. This theory got a central role in the explanation of a lot of phenomena. The relevance of these theories has been well recognized in the clarification of social phenomena and has permitted new advances in the study of social systems. Chaos theory has also been applied, particularly in the context of politics, in this area. This work goal is to make a reflection on chaos theory – and also on dynamical systems and theory of complexity – in terms of the understanding of political issues, considering some events in the political context and also considering the macro-strategic ideas of states positioning in the international stage. In this line of reasoning the "Drop of Honey Effect" metaphor is presented, somewhat analogous to that of "Butterfly Effect", and the idea that it is better suited to portray social phenomena, in particular political phenomena, is defended.

Keywords: *Chaos Theory, Politics, "Butterfly Effect", "Drop of Honey Effect"*

INTRODUCTION

Ideas inspired by Newton's thinking and the influence of scientists as Leibniz, Euler, Lagrange and even Descartes and Comte, allowed that the classical positivist model – and its deterministic features – acquired a great importance since the 18th century. It became a determinant influence in the modern scientific knowledge and has strongly supported, as Geyer (2003) stated, the idea of the existence of a universal order. The principles of order, reductionism, predictability and determinism have converted in the basis for explaining and understanding the behaviour of nature and social systems.

It was supposed to exist a mechanistic world defined by differential equations, in which causes and effects were always determined by proportional laws of behaviour. There was a linear relationship between causes and effects point of view.

A new advance, towards the existence of an uncertainty world, nonlinear and unpredictable, would come, presenting a non-proportional relationship between causes and effects. Indeed, small causes can give rise to huge consequences. Poincaré, cited in (I Font and Régis, 2006), has showed that complex behaviours could also be an output from a set of linear interacting equations.

New discoveries emerged and new theories would present, in mathematics and physics, the quantum physics and the relativity theory to give a new course to the non-linear dynamic systems (Capra, 1996, cited in I Font and Régis, 2006). In essence, it is interesting to note for example that the new discoveries did not refute Newton at all, just evolved over them revealing that many phenomena were not orderly, reducible, predictable and/or determined.

In short statements, let´s see a small mention to deterministic chaos, quantum chaos and relativistic chaos, just for a brief reference:

A) Chaos is based on the fact that small differences in the initial conditions (such as those due to rounding errors in numerical computation) yield widely diverging outcomes for chaotic systems, rendering long-term prediction impossible in general. This happens even though these systems are deterministic, meaning that their future behaviour is fully determined by their initial conditions, with no random elements involved. These systems deterministic nature does not make them predictable. This behaviour is known as deterministic chaos, or simply chaos.
B) Quantum chaos theory studies how the correspondence between quantum mechanics and classical mechanics – which is based on the solution of ordinary differential equations – works in the context of chaotic systems. It studies chaotic classical dynamic systems in terms of quantum theory, intending to study the relationship between quantum mechanics and the classical chaos.
C) Through another approach, relativistic chaos describes chaotic systems under general relativity.

THE CHAOS IDEA

Though the importance grown by chaos theory in the explanation of non-linear systems behaviour, the truth is that “chaos” is far away from being completely understood and consequently determined. Many mathematical computations and laboratory research allows to analyse and to find chaos in problems to be solved but due to the structures huge complexity this sort of problems remain without a definitive solution. With the introduction of the nonlinearity[i] idea into theoretical models, chaos would emerge in the analysis and a really very complex composition would come to be observed in the field data.

Chaos and complexity theories depict the idea that many activities reflect dynamic forms of analysis and a very complex and widespread reality, specific of complex systems, which dynamics are very hard to model and understand. These realities fall within a range of situations integrated in a broader context, which is intended to be reproduced in the theory but also to be integrated in the complex environment of their own dynamic, with complex and often chaotic features into their essence.

However, with the technologic development, it is now possible, with the help of computers, to make extremely complex calculations and to understand better the chaos occurrence.

GENERALITIES ON CHAOS THEORY

Here it is intended to study situations of chaos in politics. In order to reflect on this analysis some concepts, generally accepted in chaos theory, are now introduced. So, begin by saying that "the hidden orderly patterns in chaotic behaviour can be presented in the so-called phase space", which are abstract mathematical spaces. They are a set of structured points, normally with a high number of coordinates – each particular variable taken into account by the model is associated to an own coordinate – so that each point in this abstract space represents a complete and detailed state which the analyzed system could eventually reach. Thus, the larger the dimension – number of coordinates – of the phase space, the better will be the description of a particular state reached by the system (I Font and Régis, 2006).

A trajectory portrays the evolution of any particular system, which can be described by a chain of consecutive points in its phase space. The existence of a trajectory assumes the idea of existence of an attractor, because any trajectory of a system running on the long-term is somehow "attracted" by some points or some closed, in mathematical sense, regions within the phase space describing the system in question. There are several kinds of attractors:

- **Punctual attractor**
  One single point; the trajectory tends to a stable equilibrium.

- **Periodical attractor**
  Two or more "basins of attraction" consecutively visited by the trajectory of the system; there is a periodical oscillatory system.

- **Strange attractor**
  There is no pre-defined shape; it implies a chaotic behaviour.

[i] Nonlinear means that output is not directly proportional to input, or that a change in one variable does not produce a proportional change or reaction in the related variable(s).

Generally, a chaotic behaviour is characterized by its extreme sensitivity to the initial conditions, meaning this sensitivity that a very small perturbation of the system in an initial condition may lead it to an exponentially type divergent final state. The trajectories may behave in a very different way in neighboring points, approaching and moving away one from the other in a really unpredictable way: consider for illustration either the Lorenz' metaphor of the "Butterfly effect" or the "Drop of Honey Effect" (see Ferreira *et al*, 2014) illustrated in this work.

It is also adequate to mention the critical moments, i.e., the bifurcation points – which constantly challenge the trajectory of the system – that are positioned where the sensitivity of the system to the initial conditions is stronger. There, the chaotic nature of the system reveals itself in a more radical way, conducting the system to the so-called "limit of chaos". Up to this kind of moments, the trajectory of the system might behave in a quite predictable pattern, but once reached this bifurcation point, the prior order breaks out and the system is driven by patterns of behaviour less predictable than ever before. In other words, with nonlinear dynamic systems, the bifurcation implies a change in the system's behaviour when it is changing from one attractor to a new one (see I Font and Régis, 2006).

CHAOS IN MATHEMATICAL TERMS

As Williams (1997) says, phenomena happen over time as at discrete**,** separate or distinct, intervals[ii] or as continuously[iii]. Discrete intervals can be spaced evenly in time or irregularly in time. Continuous phenomena might be measured continuously. However, it is possible to measure them at discrete intervals[iv]. Special types of equations apply to each of those ways in which phenomena happen over time. Equations for discrete time changes are difference equations and are solved by iteration, the most of the times, or analytically. In contrast, equations based on a continuous change (continuous measurements) are differential equations. The term "flow" is often associated to differential equations[v].

Differential equations are often the most accurate mathematical way to describe a smooth continuous evolution. However, some of these equations are difficult or impossible to solve. In contrast, difference equations usually can be solved right away. Furthermore, they are often acceptable approximations of differential equations. Olsen and Degn (1985) state that difference equations are the most powerful vehicle to the understanding of chaos.

It follows a mathematical tool, through which some concepts of chaos theory are modelled, that contributes to explain the possible presence of some effects based on the idea of chaos.

[ii] Examples are the occurrence of earthquakes, rainstorms or volcanic eruptions.
[iii] Examples are air temperature and humidity or the flow of water in perennial rivers.
[iv] For example, it is possible to measure air temperature only once per hour, over many days or years.
[v] For some authors (see Bergé and Pomeau, 1984), a flow is a system of differential equations. For others (see Rasband, 1990), a flow is the solution of differential equations. Note that for the Navier–Stokes equations, that describe the motion of fluid substances, surprisingly, given their wide range of practical uses, mathematicians have not yet proven that in three dimensions' solutions always exist, or that if they do exist, then they do not contain any singularity.

So, in Berliner (1992) it is referred that non-invertibility is required to observe chaos for one-dimensional dynamic systems. Additionally, it is said "everywhere invertible maps in two or more dimensions can exhibit chaotic behaviour". The study of strange attractors shows that in the long term, as time proceeds, the trajectories of systems may become trapped in certain bounded regions of the state space of the system.

The model presented in Berliner (1992) is an example in two dimensions of the Hénon map, displaying the property of having a strange attractor.

The Hénon map appears represented by the equations:

$$x_{t+1} = 1 + y_t - a x_t^2 \tag{1}$$

And

$$y_{t+1} = b x_t , \tag{2}$$

for fixed values of a and b with $t = 0,1, \ldots$

This invertible map has strange attractors and simultaneously has strong sensitivity to initial conditions. The Hénon map, representing a transformation from $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$, Jacobian is $-b$. If $0 < b < 1$, the Hénon map contracts the domains to which it is applied. These maps are said to be dissipative. On the contrary, maps that maintain the application domain are said to be conservative.

Indeed, chaos is extremely complex and difficult to be identified in the real world, using the workable information. But, up to a certain point, it is possible to find specific mathematical relationships for problems to be solved either in computers or with laboratory research. As said before, as soon as the idea of nonlinearity was introduced into theoretical models, the existence of chaos analysed through the models was made possible. A very complex structure is observed in field data and just a simple pattern can be found and approximated theoretically; complex patterns to be got through models are much more difficult to find. In any event, it is not possible just grab a nice little set of data, apply a simple test or two, and declare "chaos" or "no chaos" (Williams, 1997). Chaos occurs in deterministic, nonlinear, dynamical systems.

The word "chaos" supposes the existence of turbulence and disorder. The predisposition to a profound change in the direction of a phenomenon generates an own force, understood as a deep change that results from small changes in the initial conditions. The chaos is - from this point of view - something extremely sensitive to the initial conditions. The sensitive dependence on initial conditions shows how a very small change at either a place or a moment in a nonlinear system can result in quite large differences to a later state in the system.

The deterministic chaos, present in many nonlinear systems, can impose fundamental limitations on the human ability for predicting behaviours. Additionally, the exploration of a big number of conditions by a single deterministic result may create the possibility of having a prospective outcome in terms of adaptation and evolution. In the context of artificial life models this has led to the notion of "life at the edge of chaos" expressing the principle that a delicate balance of chaos and order is optimal for successful evolution (Campbell and Mayer-Kress, 1997). Nevertheless, the essence of life may conduct to specific situations that sometimes bring new ones creating a new order even considering extremely difficult situations.

SOCIAL AND POLITICAL SYSTEMS AND CHAOS

The understanding of inherently nonlinear phenomena present in politics shows that it is possible to use mathematical models in the political environment analysis and socio-political issues. Moreover, when this does not happen, is yet possible to perform some kind of qualitative analysis by following the ideas of chaos theory, see I Font (2014).

In the study of social or political phenomena, the scientific object is by definition far different from the one in natural sciences. As I Font and Régis (2006) say, citing Prigogine and Nicolis (1989), social and political scientists find out that “a high degree of unpredictability of the future is the essence of the human adventure”. Some studies and research projects have assumed, in the two last decennia, that chaos theory concepts and tools are inherently part of the properties of the political science. Many studies deal with this subject by analysing situations of sensitivity to initial conditions, considering bifurcations, or entropy, for example, and use the chaos’ vocabulary to describe political behaviours and phenomena like wars, revolutions, electoral instability, or simply political problems that, on the first sight, look complex (see I Font and Régis, 2006).

It is interesting to see that often, after strategic political decisions are taken, it is very difficult to turn them back and make the decisions to be reversible. After these strategic measures are announced, the complete irreversibility of the assumed political decisions, in general, is not possible anymore and if that happens for any reason the political credibility of that government falls drastically and, since then, its fragility increases exponentially. In fact, after some courses of action are introduced it is practically impossible to reverse them. In general, future political developments result from the existence of critical moments with significant consequences in social life of people. See, for example, the case of Greece in the recent first round elections for Greek Parliament or the announcement in September and October 2012 of severe political measures in Portugal with dramatic foreseen consequences for Portuguese people.

APPROCHING POLITICAL PHENOMA WITH CHAOS METHODOLOGY

In addition to the consideration of a set of interesting situations involving chaos in politics, it is possible to typify two specific situations often seen in this area. These two evident situations can in fact result from chaos theory, in a general way, when scheming chaos systems and relating them to political science. As can be seen in I Font and Régis (2006), referring to Peled (2000), the first type encompasses systems that converge to equilibrium or a steady state, like national sentiments that often converge to a steady equilibrium. The second type concerns systems that display a stable oscillating behaviour according to a repeated pattern, like elections' cycles. The chaotic system displays an irregular oscillatory process like, for example, in countries that irregularly oscillate between anarchy, civil war and democracy.

When political phenomena are considered, it may be said that, like for the general condition in social systems, there is a high degree of unpredictability associated to the human behaviour, because in its essence, human species has a large range for unexpected actions (see Filipe and Ferreira, 2013).

In this sense, in political area, chaos theory may be applied to:

- Public organizations, as complex systems, by analysing their services and activities, by studying their equilibrium and dynamic stability, by studying the behaviour and structure of the work system. It is interesting to note particularly that, as a consequence of the utilization of chaos theory, it is possible to verify that organizations are capable of producing within themselves forces of dissipative structures most of which have self-organizing capacities that lead to new organizational entities and order. For instance, some governments' types and democracy may be considered chaotic (see I Font and Régis, 2006). This reality brings a capacity to understand how large is the possibility to build new situations, some of them with very severe consequences.
- Additionally, when studying international relations, chaos theory can be used for example in the study of peace scenarios. In I Font and Régis (2006) the importance of chaos theory is shown in this area as much as the importance of the relation between order and disorder in the emergence of peace. Many and many examples involving chaos theory in international relations area can be presented. For illustration: the Iranian revolution of 1978-1979[vi] (Farazmand, 2003); the predictions made on the post-Castro environment in Cuba[vii] (Radu, 2000); Adolf Hitler in Germany[viii] (Peled, 2000); September 2001 in the USA[ix]; Alexander in the Persian Empire[x]; the arrival of Attila to Europe[xi]; the arrival of gunpowder in Europe[xii], for example.

[vi] The spontaneous and mass revolution in Iran is considered a massive rupture of chaotic uncertainties and bifurcations into unpredictable dynamical changes in a political system.

[vii] The scenarios for the future could be based on chaotic uncertainties and bifurcations resulting into unpredictable dynamic changes in the political system.

[viii] A single man was considered the "butterfly wing "that could cause the German system to bifurcate from democracy to totalitarianism.

[ix] The tragic event of 11st September in New York brought a chaotic uncertainty to the international political and military arena.

[x] The Macedonian Alexander, endowed with great political vision, has created one of the largest empires of the ancient world, unifying the Greek state-cities and mastering the whole Eastern Antique World, with huge consequences for Humanity.

[xi] A military victory of the Chinese dynasty Han around the year 100 over a Mongolian tribe of the North (Xiongnu tribe) can be considered as the beating of the "butterfly wings" for the tragedies that would occur in medieval Europe. Indeed, the arrival of Attila and his Hunnish Army to the north of the Black Sea in the fourth century may well have been the consequence of that victory in China. This arrival would promote events with long lasting destructive effects in Europe in the middle ages. The Alliance between German and Asian tribes led to invasions and destruction throughout Europe and North Africa. It was the "Butterfly Effect" working.

[xii] The discovery of gunpowder in China may have been the initial condition for Europe leaving of the tragic situation in which it was emerged, particularly through its military expansion to the Americas, with the known consequences.

- In terms of political parties and elections, a small event during an electoral campaign can be responsible for a complete change in the final outcome. For illustration, in Portugal in 1986, in an electoral campaign in Marinha Grande (a small town), the candidate to the Presidency of the Portuguese Republic, Mário Soares, was attacked by a protester. The television showed this attack and a profound change was given by this event to the electoral results. It was the decisive moment of the turn of his first presidential campaign. He gained the election, when at the moment of the incident he had just a very small percentage of expected votes. Also being applied to political actors and parties, chaos theory can be applied to the example of the Portuguese Party CDS/PP, which is also interesting on this. After winning the party leadership in 1992, Manuel Monteiro changed the party name, adding Partido Popular. However, in 1995, Paulo Portas would assume the leadership and proposed reconciliation within the party and the return to Christian-democracy, achieving good results in the parliamentary elections. Paulo Portas has changed significantly the ideology of the party returning to Christian democracy, which allowed him to get excellent results in the elections. This phenomenon considering the ideological trajectory can be modelled as "chaotic ideological system", where a bifurcation point conducted the system to a new order.
- Examples of political systems can be also presented. For example, in the Arab region the Gulf war introduced chaos in the Arab political system. After the war, it was easy to meet small changes provoking big effects in Arab politics. The war has destabilized the system and several bifurcations were identified. In the social sphere it was possible to find oscillations between traditional patterns of stratification and modern patterns of power, privilege and influence. In the political area it was possible to find oscillations between an internal sphere where struggles of power are not softening by cultural norms and an external sphere where such struggles are bounded by cultural norms (see I Font and Régis, 2006).

THE ACTUAL WORLD GEO-STRATEGIC SITUATION

It is not simple to know where capitalism is going next as it continues to seek out new sources for rehabilitation. Some democratic states are living troubled times and new threatens arrive. Capitalism is a heterogeneous and continually dynamic process of increasingly global connection – often made through awkward and makeshift links – and those links can be surprising, not least because they often produce unexpected spatial formations which can themselves have force (Amin 2004, Bayart 2001, Moore 2004; Tsing 2005, cited in Thrift, 2008).

Chaos may reflect the true internal force that defines the disturbed system that is trying to find out new waves of consolidation, being in an unstable balance and in a precarious position facing, for instance, the new developments of European political events. The politicians and parties are discredited and public opinion develops successive manifestations of disrepute in relation to the politicians. People and public opinion is increasingly more aware of the disarray that is disturbing their lives and frustrates their expectations. The news, available around and everywhere, in all media means bring public conscience of a disordered situation, which is becoming persistent.

There is a new context for European countries that brings a state of chaotic environment, that is reflected in economic, social and political crises, in a new state of national and international context, particularly in Europe, which – in its essence – is very unfriendly for citizens in the point of view that in the last decades the context was socially and economically calmer and quieter. There is now a new reality with sudden and rapid changes, characterized by confusions and things out of control.

These new circumstances prevail now and characterize modern societies and organizations, which are based on very complex systems. As Farazmand (2003), cited in I Font and Régis (2006) says, political leaders and managers must therefore be prepared to deal with such chaotic phenomena and manage complex organizations accordingly. Indeed, chaos theory supplies tools that permit to understand better this political reality. In a certain way, there is an unpredictability of outcomes of chaotic states or systems that pose some kind of dangerous, and eventually potentially fatal, threats to individuals, groups and even to cultures. Considering the public policies, the nowadays state of countries political positioning brings some concerns about a critical point for the maintenance of the *status quo*. There is a strong complexity that needs to be understood. Complex systems theory and chaos theory unquestionably may contribute to understand it.

RECENT SITUATIONS SUPPORTING THE IDEA OF CHAOS

Indeed, in politics chaos may be evidenced in a lot of situations. Historically, simple facts with no visible significant consequences have registered considerable impacts that could not be predictable at the initial moment. Nowadays, such kind of situations continues occurring in many socio-political contexts around the world. The “Arabian Spring” is an example of how the “Butterfly Effect” can be found when causing a wide spread regional political reform in the political regimes of some countries in that geographical area. The “flapping of the butterfly wings” may be represented by the immolation by fire of a Tunisian salesman that was the starting point for the regime change in Tunisia first and then the contagion to Egypt and Libya. The consequences would be seen as well in Syria where a civil war is yet in course. The “Butterfly Effect” could also be named as the “Drop of Honey Effect”[xiii], which is much more suggestive for socio-political events, from the tale written by the Armenian poet Hovanés Tumanian (1869-1923).

[xiii] On a warm afternoon, on the second floor of a splendid palace that overlooked the market place of the city, sat a king and his minister. While the king was eating some puffed rice on honey, he looked over his land with satisfaction. What a prosperous city he ruled. What a magnificent city.
As he was daydreaming, a little drop of honey dripped from his puffed rice onto the window ledge.
The minister was about to call a servant to wipe up the honey, when the king waved a hand to stop him. “Don’t bother, it’s only a little drop of honey, it’s not our problem.”

Giving another example and considering the political situation in Greece in May 2012, a new stage came to be studied for Greek, European and World economy. The political *status quo* was broken in Greece: a new party took an advantage that it had never had. In fact, an emergent crisis in Greece was severely felt after the Greece-*Troika* agreement. Throughout this Program, Greece has to respect an austerity program in order to put national budgeting at acceptable levels and is complied to obey the agreement that is conducting Greek people to severe self well being sacrificing. This situation led Greeks to vote in favor of a new situation in the first round elections. Although the second round kept the *status quo* in the political situation, the truth is that this could become an entire new situation that could impose a new socio-economic condition to European Union and to the World that could threat the world economic stability. The possible bankruptcy in Greece was tormenting world leaders; a new *status quo* was being prepared for Europe with considerable implications for the whole world. This scenario was adjusted after the second round elections, but the alert was there.

THE DISSIPATIVE EFFECT ON POLITICS APPROACHED MATHEMATICALLY

Inspired in the model in Berliner (1992) seen above, it is possible now to suggest one for economics politics in the fisheries area[xiv].

So, if a general situation is considered, the following equations may represent a system in which fish stocks, at time t, are given by x_t and catches by y_t. The model is as follows:

The minister watched the drop of honey slowly trickle down the window ledge and land on the street below.
Soon, a buzzing fly landed on the sweet drop of honey.
A nearby lizard shot out its long tongue and caught the fly.
The lizard was taken by surprise when a cat leapt on it.
The cat was pounced on by its worst enemy the dog that had broken free from its chain.
Meeowing and barking erupted from the street below the King and his minister. The minister was about to call a servant to go and deal with the brawling cat and dog when the king said, "Relax, the cat and dog belong to the market people. We shouldn't interfere. It's not our problem."
The cat's owner was horrified to see her cat being attacked by the big bully of a dog and started whacking the dog with her broom. The dog's owner was horrified to see her dog being attacked by the big bully of a cat and started whacking the cat with her broom.
Soon, people started coming out from their stalls and houses to see what all the screaming and shouting was about. Seeing their friend's cat being attacked, they joined in berating the dog and its owner. Others, seeing their friend's dog being attacked by the cat, also joined in. Very quickly, the shouting became violent and a fight broke out in the street.
The worried minister turned to the King but his only comment was, "Not our problem. Here, have some more puffed rice and honey." The king and his adviser ate as they watched the fray below.
Soon the police were called in to break up the fight, but the people were so angry, each side convinced that they were right, (right about what, they couldn't remember). They started attacking the policemen. The fight rapidly broke out into a full-scale riot.
The king eyed the minister and said, "I know what you are thinking, but the army will handle it. Besides, this is not our problem."
The riot swiftly escalated into a civil war with looting and destruction all over the city. Buildings were set alight and by nightfall, the magnificent city was reduced to a pile of smoking ashes. The king and his minister stood spellbound rooted to the spot where they had been watching all day. Their mouths were hanging open in horror.
"Oh…" said the king quietly, "maybe the little drop of honey WAS our problem." (freely adapted from the tale of Hovanés Tumanian).

[xiv] And also, evidently, in other reproducing and harvesting natural resources areas.

$$x_{t+1} = F(x_t) - y_t$$

and

$$y_{t+1} = bx_t . \quad (3)$$

It is a generalization of Hénon model. The Jacobian is *b*. As y_{t+1} is a portion of x_t, $0 < b < 1$. So, it is a dissipative model and the values of x_t are restricted to a bounded domain.

Considering the particular case below:

$$x_{t+1} = x_t - y_t$$

and

$$y_{t+1} = bx_t . \quad (4)$$

So,

$$x_{t+2} = x_{t+1} - y_{t+1}$$

and

$$x_{t+2} - x_{t+1} + bx_t = 0 . \quad (5)$$

Now, after solving the characteristic equation associated to the difference equation (see Ferreira and Menezes, 1992) it is obtained:

$k = \frac{1+\sqrt{1-4b}}{2}$ or $k = \frac{1-\sqrt{1-4b}}{2}$; calling $\Delta = 1-4b$ and being $0 < b < 1$, it results $-3 < \Delta < 1$.

So, $0 < \Delta < 1$ if $0 < b < \frac{1}{4}$ and $-3 < \Delta < 0$ if $\frac{1}{4} < b < 1$, being $\Delta = 0$ when $b = \frac{1}{4}$.

Consequently for $0 < b < \frac{1}{4}$,

$$x_t = A_1 \left(\frac{1+\sqrt{1-4b}}{2} \right)^t + A_2 \left(\frac{1-\sqrt{1-4b}}{2} \right)^t \quad (6).$$

And for $b = \frac{1}{4}$,

$$x_t = \left(A_1 + A_2 t \right) \left(\frac{1}{2} \right)^t \quad (7)$$

Finally, for $\frac{1}{4} < b < 1$

$$x_t = \left(\sqrt{b}\right)^t \left[A_1 \cos\left(\left(\arccos\frac{1}{2\sqrt{b}}\right)t\right) + A_2 sen\left(\left(\arccos\frac{1}{2\sqrt{b}}\right)t\right)\right] \quad (8)$$

In these solutions, A_1 and A_2 are real constants.

Note that the bases of t powers are always between 0 and 1. So, $\lim_{t\to\infty} x_t = 0$ and whatever the value of b, the dissipative effect is real, even leading to the extinction of this species. Of course, this is evident according to the hypotheses of this particular situation of the model.

Concluding this approach, the model does not allow to obtain in general simple explicit solutions. But, of course, with simple computational tools it is possible to obtain recursively concrete time series solutions after establishing the initial value x_0 and to check the dissipative effect. Quite the opposite, it is extremely simple to predict the behavior of the system by interpreting the parameters values.

Additionally a new example may be presented for politics, in general, considering the political credibility.

Call x_t the political credibility, of a politician or of a party measured, for instance, in number of votes, or in the number of chamber's members, or even in money, in the year t; and consider b the credibility rate, $-1 \le b \le 1$.

It is admissible that in the year t+1, $x_{t+1} = x_t + bx_t$, that is: in a certain year the political credibility is the one of the former year plus, or minus, a part of it. So:

$$x_{t+1} - (1+b)x_t = 0. \quad (9)$$

Solving this difference equation (see Ferreira and Menezes, 1992) it is obtained[xv]:

$$x_t = x_0(1+b)^t, b \neq 0 \text{ and } x_t = x_0, b = 0. \quad (10)$$

Then, according to this model, if the credibility rate is null the political credibility is kept unchanged, assuming always the initial value. If $0 < b \le 1$, the political credibility follows an increasing exponential path. If $-1 < b < 0$, the political credibility follows a decreasing exponential path converging to 0. Finally, if $b = -1$, x_t is permanently null. Evidently, values like $-1 \le b < 0$ define political credibility paths that may lead to people's chaotic behaviours.

CONCLUDING REMARKS

Was tried in this text, and it was mainly achieved, to show that in politics, chaos may be evidenced for innumerable situations. Historically, there are a lot of simple facts, considered insignificant in the moment for the consequences they had, that in a completely unexpected way gave rise to huge impacts that could not

[xv]Evidently, this is the compound interest capitalization formula, at interest rate b used for financial purposes.

be predicted, or even guessed, at the its occurrence initial moment. Indeed, are situations for which the output is not directly proportional to the input.

Nowadays, such facts continue happening in many socio-political contexts around the world. It is at least strange that the simple – despite the greatness of the personal sacrifice – immolation by fire of a Tunisian salesman was the starting point for the regime change in Tunisia first and then the contagion to Egypt and Libya. The consequences would then be seen as well in Syria where a bloody civil war (?) is still in course.

The "Arabian Spring" was presented in this work as an example of the way how the "Butterfly Effect" can be found when causing a wide spread regional political reform in the political regimes of some countries in that geographical area. The "flapping of the butterfly wings" may be represented by that immolation by fire of a Tunisian salesman. It is here suggested the expression, inspired in the wonderful tale written by the Armenian poet Hovanés Tumanian (1869-1923), "Drop of Honey Effect" use instead of "Butterfly Effect" when dealing with socio-political events. The term "Butterfly Effect" seems more adequate when dealing with physical and natural events. Really, the "Drop of Honey Effect" seems much more adequate and accurate to evidence chaos in social and political situations.

The 1986 Presidency of the Portuguese Republic electoral campaign case is an example of how an insignificant incident produces a complete change in the electoral results. This is not, at least completely, a case of "small input-great output" but instead of "small input-reverse output", which in its turn evidences the presence of chaos as well. Of course in this situation, for the candidate Mário Soares himself, the "Drop of Honey Effect" was the attack perpetrated against him, despite of its bitterness.

In the end of this article two mathematical models, with difference equations, were presented. They contribute to identify possible chaotic situations, in politics, through the values of the models' parameters. The more accurate is the evaluation of these parameters, the more is the usefulness of each of the models.

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