OPERATIONS PRE-CONTINUOUS MAPPINGS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we define the operations mappings in topological spaces such as (γ^*, β)-pre-continuous, (γ^*, β^*)-pre-continuous, (γ, β^*)-pre-open, (γ^*, β^*)-pre-open, (γ, β^*)-pre-closed, (γ^*, β^*)-pre-closed and (γ^*, β^*)-pre-homeomorphism and study some their basic properties.

Keywords: *(γ^*, β)-pre-continuous, (γ^*, β^*)-pre-continuous, (γ, β^*)-pre-open, (γ^*, β^*)-pre-open, (γ, β^*)-pre-closed, (γ^*, β^*)-pre-closed, γ^*-pre-open, β^*-pre-open*

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INTRODUCTION

Mashhour et al.[6] and Andrijevic[1, 2] introduced the concept of pre-open sets and semi-pre-open sets respectively. Kasahara[3] defined the concept of operations α on topological spaces. Ogata[7, 8] called the operations α (resp. α-closed set) as γ-operations (resp. γ-closed set) and introduced the notion of τ_γ which is the collection of γ-open sets in topological spaces. Sai Sundara Krishnan and Balachandran[9] initiated the concept of γ-pre-open sets and studied the separation axioms using γ-pre-open sets. Further, they generated a topology $\tau_{\gamma p}$ using γ-pre-open sets. Sai Sundara Krishnan et al.[10] obtained the concept of γ^*-pre-open sets and γ^*-semi-pre-open sets in topological spaces and investigated some basic properties. D. Saravanakumar et al.[11, 12] generated the idea of operations mappings in topological spaces.

In this paper in section 3, we created the concept of operations approaches continuous mappings in topological spaces such as (γ^*, β)-pre-continuous, (γ^*, β^*)-pre- continuous. Also, we investigated some of their essential properties through the γ^*-pre-open, γ^*-pre-closed and γ^*-pre-derived sets. In section 4, we obtained the idea of operations open, closed mappings such as (γ^*, β^*)-pre-open, (γ^*, β^*)-pre-closed and studied some of their important properties. Moreover, we shows that every (γ^*, β^*)-pre-continuous (γ^*, β^*)-pre-closed image of γ^*-pg.closed set is β^*-pg.closed. In addition, we proved that every (γ^*, β^*)-pre-continuous (γ^*, β^*)-pre-closed inverse image of β^*-pg.closed set is γ^*-pg.closed.

PRELIMINARIES

An operation γ[3] on the topology τ is a mapping from τ into the power set $P(X)$ of X such that $V \subseteq V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at V. It is denoted by $\gamma: \tau \to P(X)$. A subset A of X is γ-open[7], if for each $x \in A$, there exists an open neighborhood U such that $x \in U$ and $U^\gamma \subseteq A$. Its complement is called γ-closed and τ_γ denotes set of all γ-open sets in X. For a subset A of X, γ-interior[7] of A is $int_\gamma(A) = \{x \in A : x \in N \in \tau$ and $N^\gamma \subseteq A$ for some $N\}$ and γ-closure[7] of A is $cl_\gamma(A) = \{x \in X : x \in U \in \tau$ and $U^\gamma \cap A \neq \emptyset$ for all $U\}$. An operation γ on τ is regular[7], if for any open neighborhoods U, V of each $x \in X$, there exists an open neighborhood W of x such that $U^\gamma \cap V^\gamma \supseteq W^\gamma$; open[7], if for every neighborhood U of each $x \in X$, there exists a γ-open set B such that $x \in B$ and $U^\gamma \supseteq B$. A space X is γ-regular[7], if for each $x \in X$ and for each open neighborhood V of x, there exists an open neighborhood U of x such that $U^\gamma \subseteq V$. A subset A of X is γ^*-dense (resp. γ^*-nowhere dense, γ^*-pre-open)[10], if $cl_\gamma(A) = X$ (resp. $int_\gamma(cl_\gamma(A)) = \emptyset$, $A \subseteq int_\gamma(cl_\gamma(A))$). The set of all γ^*-pre-open sets is denoted by $PO_{\gamma^*}(X)$. A is γ^*-pre-closed[10] in X if and only if $X - A$ is γ^*-pre-open in X. A is γ^*-pre-clopen[10], if A is both γ^*-pre-open and γ^*-pre-closed in X. For a subset A of X, γ^*-pre-interior[10] of A is $pint_{\gamma^*}(A) = \cup\{U : U \in PO_{\gamma^*}(X)$ and $U \subseteq A\}$and γ^*-pre-closure[10] of A is $pcl_{\gamma^*}(A) = \cap\{F : X - F \in PO_{\gamma^*}(X)$ and $A \subseteq F\}$. A space X is γ^*-submaximal[10], if every γ^*-dense set of X is γ-open in X. A subset A of X is γ^*-pg.closed if $pcl_{\gamma^*}(A) \subseteq U$ whenever $A \subseteq U$ and U is γ^*-pre-open. A space X is γ^*-pre-T_0[10] if for each distinct points $x, y \in X$, there exists a $U \in PO_{\gamma^*}(X)$ such that $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$. A space X is γ^*-pre-T_1[10] if for each distinct points $x, y \in X$, there exists $U, V \in PO_{\gamma^*}(X)$ such that $x \in U$, $y \notin U$, $x \notin V$ and $y \in V$. A space X is γ^*-pre-T_2[10] if for each distinct points $x, y \in X$, there exists $U, V \in PO_{\gamma^*}(X)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. A space X is γ^*-pre-$T_{\frac{1}{2}}$[10] if for each γ^*-pg.closed set of X is γ^*-pre-closed. A mapping $f : X \to Y$ is (γ, β)-irresolute[5] if for any β-open set B of Y, $f^{-1}(B)$ is γ-open in X.

Proposition 2.1.[10] Every γ^*-pre-closed set is γ^*-pg.closed set. But the converse need not be true.

Throughout this paper let X, Y and Z be three topological spaces and operations $\gamma: \tau \to P(X)$, $\beta: \sigma \to P(Y)$ and $\rho: \eta \to P(Z)$ on topologies τ, σ and η respectively. Here $PO_{\gamma^*}(X)$, $PO_{\beta^*}(Y)$ and $PO_{\rho^*}(Z)$ are denotes the family of γ^*-pre-open sets, β^*-pre-open sets and ρ^*-pre-open sets respectively.

(γ^*, β^*)-PRE-CONTINUOUS MAPPINGS

Definition 3.1. A mapping $f: X \to Y$ is said to be (γ^*, β)-pre-continuous (resp. (γ^*, β^*)-pre-continuous) if $f^{-1}(O)$ is γ^*-pre-open in X whenever O is β-open (resp. β^*-pre-open) in Y.

Remark 3.1. (i) Every (γ^*, β^*)-pre-continuous mapping is (γ^*, β)-pre-continuous. But the converse need not be true.

Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{\emptyset, Y, \{1\}, \{1, 3\}\}$ and define operations $\gamma: \tau \to P(X)$ and $\beta: \sigma \to P(Y)$ by
$\gamma(A) = \begin{cases} cl(A) & \text{if } A = \{a,c\} \\ A & \text{if } A \neq \{a,c\} \end{cases}$ for every $A \in \tau$ and $\beta(A) = \begin{cases} A & \text{if } 3 \in A \\ A \cup \{3\} & \text{if } 3 \notin A \end{cases}$ for every $A \in \sigma$ respectively.

Define $f: X \to Y$ by $f(a) = 3$, $f(b) = 2$ and $f(c) = 1$. Then f is (γ^*, β)-pre-continuous. Also $f^{-1}(\{1\}) = \{c\}$ is not γ^*-pre-open in X for the β^*-pre-open set $\{1\}$ of Y. Hence f is not (γ^*, β^*)-pre-continuous.

(ii) The concepts of (γ^*, β^*)-pre-continuous and (γ, β)-irresolute mappings are independent.

Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{\emptyset, Y, \{2\}, \{2, 3\}\}$ and define operations $\gamma: \tau \to P(X)$ and $\beta: \sigma \to P(Y)$ by
$\gamma(A) = \begin{cases} cl(A) & \text{if } A = \{c\} \\ A & \text{if } A \neq \{c\} \end{cases}$ for every $A \in \tau$ and $\beta(A) = \begin{cases} cl(A) & \text{if } A = \{2\} \\ A & \text{if } A \neq \{2\} \end{cases}$ for every $A \in \sigma$ respectively.

Define $f: X \to Y$ by $f(a) = 2$, $f(b) = 1$ and $f(c) = 3$. Then f is (γ, β)-irresolute. Also $f^{-1}(\{2\}) = \{a\}$ is not γ^*-pre-open in X for the β^*-pre-open set $\{2\}$ of Y. Hence f is not (γ^*, β^*)-pre-continuous.

Also, consider $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$ and $\sigma = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$ and define operations $\gamma: \tau \to P(X)$ and $\beta: \sigma \to P(Y)$ by
$\gamma(A) = \begin{cases} A & \text{if } c \in A \\ A \cup \{c\} & \text{if } c \notin A \end{cases}$ for every $A \in \tau$ and $\beta(A) = \begin{cases} A \cup \{3\} & \text{if } A = \{1\} \\ A & \text{if } A \neq \{1\} \end{cases}$ for every $A \in \sigma$ respectively.

Define $f: X \to Y$ by $f(a) = 1$, $f(b) = 3$ and $f(c) = 2$. Then f is (γ^*, β^*)-pre-continuous. But $f^{-1}(\{2\}) = \{c\}$ is not γ-open in X for the β-open set $\{2\}$ of Y. By Proposition 4.13[7], f is not (γ, β)-irresolute.

(iii) Every (γ,β)-irresolute mapping is (γ^*,β)-pre-continuous. But the converse need not be true.

Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{\emptyset, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$ and define operations $\gamma: \tau \to P(X)$ and $\beta: \sigma \to P(Y)$ by
$\gamma(A) = \begin{cases} A & if\ A=\{a,b\} \\ cl(A) & if\ A\neq\{a,b\} \end{cases}$ for every $A \in \tau$ and $\beta(A) = \begin{cases} cl(A) & if\ A=\{1,3\} \\ A & if\ A\neq\{1,3\} \end{cases}$ for every $A \in \sigma$ respectively.

Define $f: X \to Y$ by $f(a) = 1$, $f(b) = 2$ and $f(c) = 3$. Then f is (γ^*,β)-pre-continuous. Also $f^{-1}(\{2\}) = \{b\}$ is not γ-open in X for the β-open set $\{2\}$ of Y. Hence f is not (γ,β)-irresolute.

(iv) If X, Y are γ-regular space and β-regular space respectively, then the concepts of (γ^*,β^*)-pre-continuous and pre-continuous mappings coincide.

Theorem 3.1. Let $f: X \to Y$ be a mapping. Then the following statements are equivalent:

(i) f is (γ^*,β)-pre-continuous;

(ii) for each $x \in X$ and each β-open set $V \subseteq Y$ containing $f(x)$, there exists $W \in PO_{\gamma^*}(X)$ such that $x \in W$, $f(W) \subseteq V$;

(iii) the inverse image of each β-closed set in Y is γ^*-pre-closed in X.

Proof. (i) $\Rightarrow$ (ii). Let $x \in X$ and V be any β-open set of Y containing $f(x)$. Set $W = f^{-1}(V)$, then by Definition 3.1, W is a γ^*-pre-open set containing x and $f(W) = f(f^{-1}(V)) \subseteq V$.

(ii) $\Rightarrow$ (iii). Let F be a β-closed set of Y. Set $V = Y - F$, then V is β-open in Y. Let $x \in f^{-1}(V)$, by (ii), there exists a γ^*-pre-open set W of X containing x such that $f(W) \subseteq V$. Thus, we obtain that $x \in W \subseteq int_\gamma(cl_\gamma(W)) \subseteq int_\gamma(cl_\gamma(f^{-1}(V)))$ and hence $f^{-1}(V) \subseteq int_\gamma(cl_\gamma(f^{-1}(V)))$. This shows that $f^{-1}(V)$ is γ^*-pre-open in X. Hence $f^{-1}(F) = X - f^{-1}(Y - F) = X - f^{-1}(V)$ is γ^*-pre-closed in X.

(iii) $\Rightarrow$ (i). Let B be a β-open set in Y. Then $F = Y - B$ is β-closed in Y. By (iii), $f^{-1}(F)$ is γ^*-pre-closed in X. Hence $f^{-1}(B) = X - f^{-1}(Y - B) = X - f^{-1}(F)$ is γ^*-pre-open in X.

Theorem 3.2. Let $f: X \to Y$ be a mapping and $\beta: \sigma \to P(Y)$ be an open operation on σ. Then the following statements are equivalent:

(i) f is (γ^*,β)-pre-continuous;

(ii) $cl_\gamma(int_\gamma(f^{-1}(B))) \subseteq f^{-1}(cl_\beta(B))$ for each $B \subseteq Y$;

(iii) $f(cl_\gamma(int_\gamma(A))) \subseteq cl_\beta(f(A))$ for each $A \subseteq X$.

Proof. Follows from the Definition 3.1 and Theorem 3.1(iii).

Theorem 3.3. Let $f: X \to Y$ be a (γ, β)-irresolute mapping and $\beta: \sigma \to P(Y)$ be an open operation on σ. Then (i) $f(cl_\gamma(U)) \subseteq cl_\beta(f(U))$ for each γ-open set U in X;

(ii) $cl_\gamma(f^{-1}(V)) \subseteq f^{-1}(cl_\beta(V))$ for each β-open set V in Y.

Proof. Follows from the Remark 3.1 and Theorem 3.2.

Theorem 3.4. If $f: X \to Y$ is a (γ^*, β)-pre-continuous mapping and X_0 is a γ-open subset of X, then the restriction $f|X_0$: $X_0 \to Y$ is (γ^*, β)-pre-continuous, where $\gamma: \tau \to P(X)$ is a regular operation on τ.

Proof. Follows from the Definition 3.1 and Theorem 2.5[10].

Theorem 3.5. Let X be a topological space, $\gamma: \tau \to P(X)$ be a regular operation on τ and $\{V_k : k \in J\}$ a cover of X by γ-open sets of X. A mapping $f: X \to Y$ is (γ^*, β)-pre-continuous if and only if the restriction $f|V_k$: $V_k \to Y$ is (γ^*, β)-pre-continuous for each $k \in J$.

Proof. Follows from the Theorems 2.1[10] and 3.4.

Definition 3.2. (i) Let X be a topological space and $\gamma: \tau \to P(X)$ be an operation on τ. A subset A of a space X is said to be a γ^*-pre-neighborhood of a point $x \in X$ if there exists a γ^*-pre-open set U such that $x \in U \subseteq A$.

Note that γ^*-pre-neighborhood of x may be replaced by γ^*-pre-open neighborhood of x.

(ii) Let X be a space. $A \subseteq X$ and $p \in X$. Then p is called a γ^*-pre-limit point of A if $U \cap (A - \{p\}) \neq \emptyset$ for any γ^*-pre-open set U containing p. The set of all γ^*-pre-limit points of A is called a γ^*-pre-derived set of A and is denoted by $pd_{\gamma^*}(A)$. Clearly if $A \subseteq B$ then $pd_{\gamma^*}(A) \subseteq pd_{\gamma^*}(B)$.

Remark 3.2. From the Definition 3.2(ii), it follows that p is a γ^*-pre-limit point of A if and only if $p \in pcl_{\gamma^*}(A - \{p\})$.

Theorem 3.6. For any A, $B \subseteq X$, the γ^*-pre-derived sets have the following properties:

(i) $pcl_{\gamma^*}(A) \supseteq A \cup pd_{\gamma^*}(A)$;
(ii) $\cup_i pd_{\gamma^*}(A_i) = pd_{\gamma^*}(\cup_i A_i)$;
(iii) $pd_{\gamma^*}(pd_{\gamma^*}(A)) \subseteq pd_{\gamma^*}(A)$;
(iv) $pcl_{\gamma^*}(pd_{\gamma^*}(A)) = pd_{\gamma^*}(A)$.

Proof. Follows from the Definition 3.2(ii) and Remark 3.2.

Theorem 3.7. Let $f: X \to Y$ be a mapping. Then the following statements are equivalent:

(i) f is (γ^*, β^*)-pre-continuous;

(ii) for each x in X, the inverse of every β^*-pre-neighborhood of $f(x)$ is a γ^*-pre-neighborhood of x;

(iii) for each point x in X and each β^*-pre-neighborhood B of $f(x)$, there is a γ^*-pre-neighborhood A of x such that $f(A) \subseteq B$;

(iv) for each $x \in X$ and each β^*-pre-open set B of $f(x)$, there is a γ^*-pre-open set A of x such that $f(A) \subseteq B$;

(v) $f(pcl_{\gamma^*}(A)) \subseteq pcl_{\beta^*}(f(A))$ holds for every subset A of X;

(vi) for any β^*-pre-closed set H of Y, $f^{-1}(H)$ is γ^*-pre-closed in X.

Proof. (i) $\Rightarrow$ (ii). Let $x \in X$ and B be a β^*-pre-neighborhood of $f(x)$. By Definition 3.2(i), there exists $V \in PO_{\beta^*}(Y)$ such that $f(x) \in V \subseteq B$. This implies that $x \in f^{-1}(V) \subseteq f^{-1}(B)$. Since f is (γ^*, β^*)-pre-continuous, so $f^{-1}(V) \in PO_{\gamma^*}(X)$. Hence $f^{-1}(B)$ is a γ^*-pre-neighborhood of x.

(ii) $\Rightarrow$ (i). Let $B \in PO_{\beta^*}(Y)$. Put $A = f^{-1}(B)$. Let $x \in A$. Then $f(x) \in B$. Clearly, B (being β^*-pre-open) is a β^*-pre-neighborhood of $f(x)$. By (ii), $A = f^{-1}(B)$ is a γ^*-pre-neighborhood of x. Hence by Definition 3.2(i), there exists $A_x \in PO_{\gamma^*}(X)$ such that $x \in A_x \subseteq A$. This implies that $A = \cup_{x \in A} A_x$. By Theorem 2.1[10], A is γ^*-pre-open in X. Therefore f is (γ^*, β^*)-pre-continuous.

(i) $\Rightarrow$ (iii). Let $x \in X$ and B be a β^*-pre-neighborhood of $f(x)$. Then, there exists $O_{f(x)} \in PO_{\beta^*}(Y)$ such that $f(x) \in O_{f(x)} \subseteq B$. It follows that $x \in f^{-1}(O_{f(x)}) \subseteq f^{-1}(B)$. By (i), $f^{-1}(O_{f(x)}) \in PO_{\gamma^*}(X)$. Let $A = f^{-1}(B)$. Then it follows that A is γ^*-pre-neighborhood of x and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) $\Rightarrow$ (i). Let $U \in PO_{\beta^*}(Y)$. Take $W = f^{-1}(U)$. Let $x \in W$. Then $f(x) \in U$. Thus U is a β^*-pre-neighborhood of $f(x)$. By (iii), there exists a γ^*-pre-neighborhood V_x of x such that $f(V_x) \subseteq U$. Thus it follows that $x \in V_x \subseteq f^{-1}(f(V_x)) \subseteq f^{-1}(U) = W$. Since V_x is a γ^*-pre-neighborhood of x, which implies that there exists a $W_x \in PO_{\gamma^*}(X)$ such that $x \in W_x \subseteq W$. This implies that $W = \cup_{x \in W} W_x$. By Theorem 2.1[10], W is γ^*-pre-open in X. Thus f is (γ^*, β^*)-pre-continuous.

(iii) $\Rightarrow$ (iv). We may replaced the γ^*-pre-neighborhood of x as γ^*-pre-open neighborhood of x in condition (iii). Straightforward.

(iv) $\Rightarrow$ (v). Let $y \in f(pcl_{\gamma^*}(A))$ and V be any β^*-pre-open set containing y. Then, there exists a point $x \in X$ and a γ^*-pre-open set U such that $x \in U$ with $f(x) = y$ and $f(U) \subseteq V$. Since $x \in pcl_{\gamma^*}(A)$, we have that $U \cap A \neq \emptyset$ and hence $\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. This implies that $y \in pcl_{\beta^*}(f(A))$. Therefore, we have that $f(pcl_{\gamma^*}(A)) \subseteq pcl_{\beta^*}(f(A))$.

(v) $\Rightarrow$ (vi). Let H be a β^*-pre-closed set in Y. Then $pcl_{\beta^*}(H) = H$. By (v), $f(pcl_{\gamma^*}(f^{-1}(H))) \subseteq pcl_{\beta^*}(f(f^{-1}(H))) \subseteq pcl_{\beta^*}(H) = H$ holds. Therefore

$pcl_{\gamma^*}(f^{-1}(H)) \subseteq f^{-1}(H)$ and thus $f^{-1}(H) = pcl_{\gamma^*}(f^{-1}(H))$. Hence $f^{-1}(H)$ is γ^*-pre-closed in X.

(vi) $\Rightarrow$ (i). Let B be a β^*-pre-open set in Y. We take $H = Y - B$. Then H is β^*-pre-closed in Y. By (iv), $f^{-1}(H)$ is γ^*-pre-closed in X. Hence $f^{-1}(B) = X - f^{-1}(Y - B) = X - f^{-1}(H)$ is γ^*-pre-open in X.

Theorem 3.8. A mapping $f: X \to Y$ is (γ^*, β^*)-pre-continuous if and only if $f(pd_{\gamma^*}(A)) \subseteq pcl_{\beta^*}(f(A))$, for all $A \subseteq X$.

Proof. Let $f: X \to Y$ be (γ^*, β^*)-pre-continuous. Let $A \subseteq X$ and $x \in pd_{\gamma^*}(A)$. Assume that $f(x) \notin f(A)$ and let V denote a β^*-pre-neighborhood of $f(x)$. Since f is (γ^*, β^*)-pre-continuous, so by Theorem 3.7(iii), there exists a γ^*-pre-neighborhood U of x such that $f(U) \subseteq V$. From $x \in pd_{\gamma^*}(A)$, it follows that $U \cap A \neq \emptyset$ there exists, therefore, at least one element $a \in U \cap A$ such that $f(a) \in f(A)$ and $f(a) \in V$. Since $f(x) \notin f(A)$, we have that $f(a) \neq f(x)$. Thus every β^*-pre-neighborhood of $f(x)$ contains an element $f(a)$ of $f(A)$ different from $f(x)$. Consequently, $f(x) \in pd_{\beta^*}(f(A))$. Conversely, suppose that f is not (γ^*, β^*)-pre-continuous. Then by Theorem 3.7(iii), there exists $x \in X$ and a β^*-pre-neighborhood V of $f(x)$ such that every γ^*-pre-neighborhood U of x contains at least one element $a \in U$ for which $f(a) \notin V$. Put $A = \{a \in X : f(a) \notin V\}$. Since $f(x) \in V$, therefore $x \notin A$ and hence $f(x) \notin f(A)$. Since $f(A) \cap (V - \{f(x)\}) = \emptyset$, therefore $f(x) \notin pd_{\beta^*}(f(A))$. It follows that $f(x) \in f(pd_{\gamma^*}(A)) - (f(A) \cup pd_{\beta^*}(f(A))) \neq \emptyset$, which is a contradiction to the given condition.

Theorem 3.9. Let $f: X \to Y$ be one-to-one mapping. Then f is (γ^*, β^*)-pre-continuous if and only if $f(pd_{\gamma^*}(A)) \subseteq pd_{\beta^*}(f(A))$, for all $A \subseteq X$.

Proof. Let $A \subseteq X$, $x \in pd_{\gamma^*}(A)$ and V be a β^*-pre-neighborhood of $f(x)$. Since f is (γ^*, β^*)-pre-continuous, then by Theorem 3.7(iii), there exists a γ^*-pre-neighborhood U of x such that $f(U) \subseteq V$. But $x \in pd_{\gamma^*}(A)$ gives there exists an element $a \in U \cap A$ such that $a \neq x$. Clearly $f(a) \in f(A)$ and since f is one-to-one, $f(a) \neq f(x)$. Thus every β^*-pre-neighborhood V of $f(x)$ contains an element $f(a)$ of $f(A)$ different from $f(x)$. Consequently, $f(x) \in pd_{\beta^*}(f(A))$. Therefore, $f(pd_{\gamma^*}(A)) \subseteq pd_{\beta^*}(f(A))$. Converse follows from the Theorem 3.8.

Theorem 3.10. Let $f: X \to Y$ be a (γ^*, β^*)-pre-continuous and injective. If Y is β^*-pre-T_2 (resp. β^*-pre-T_1), then X is γ^*-pre-T_2 (resp. γ^*-pre-T_1).

Proof. Suppose Y is β^*-pre-T_2. Let x and y be two distinct points of X. Then, there exists two β^*-pre-open sets U and V such that $f(x) \in U$, $f(y) \in V$ and $U \cap V = \emptyset$. Since f is (γ^*, β^*)-pre-continuous, for U and V, there exists two γ^*-pre-open sets W and S such that $x \in W$ and $y \in S$, $f(W) \subseteq U$ and $f(S) \subseteq V$, implies that

$W \cap S = \emptyset$. Hence X is γ^*-pre-T_2. In similar way one can prove that X is γ^*-pre-T_1 whenever Y is β^*-pre-T_1.

(γ^*, β^*)-PRE-OPEN MAPPINGS

Definition 4.1. A mapping $f: X \to Y$ is said to be (γ, β^*)-pre-open (resp. (γ, β^*)-pre-closed, (γ^*, β^*)-pre-open, (γ^*, β^*)-pre-closed) if $f(O')$ is β^*-pre-open (resp. β^*-pre-closed, β^*-pre-open, β^*-pre-closed) in Y whenever O' is γ-open (resp. γ-closed, γ^*-pre-open, γ^*-pre-closed) in X.

Remark 4.1. (i) Every (γ^*, β^*)-pre-open(closed) mapping is (γ, β^*)-pre-open(closed). But the converse need not be true.

Note that if $f: X \to Y$ is (γ^*, β^*)-pre-open(closed) and $g: Y \to Z$ is (β^*, ρ^*)-pre- open(closed), then the composition $g o f: X \to Z$ is a (γ^*, ρ^*)-pre- open(closed) mapping.

Theorem 4.1. Let $f: X \to Y$ be a mapping. Then the following statements are equivalent:

(i) f is (γ, β^*)-pre-open;

(ii) for each $x \in X$ and each γ-neighborhood U of x, there exists a β^*-pre-open set V of Y such that $f(x) \in V \subseteq f(U)$;

(iii) for each subset $W \subseteq Y$ and each γ-closed set F of X containing $f^{-1}(W)$, there exists a β^*-pre-closed set H of Y such that $W \subseteq H$ and $f^{-1}(H) \subseteq F$.

Proof. (i) $\Rightarrow$ (ii). Suppose that f is a (γ, β^*)-pre-open mapping. For each $x \in X$ and each γ-neighborhood U of x, there exists a γ-open set U_0 such that $x \in U_0 \subseteq U$. Since f is (γ, β^*)-pre-open, $V = f(U_0)$ is β^*-pre-open and $f(x) \in V \subseteq f(U)$.

(ii) $\Rightarrow$ (i). Let U be a γ-open set of X. For each $x \in U$, there exists a β^*-pre-open set $V_{f(x)}$ such that $f(x) \in V_{f(x)} \subseteq f(U)$. Therefore, $f(U) = \cup\{V_{f(x)} : x \in U\}$and hence by Theorem 2.1[10], $f(U)$ is β^*-pre-open. This shows that f is (γ, β^*)-pre-open.

(i) $\Rightarrow$ (iii). Let $H = Y - f(X - F)$. Since $f^{-1}(W) \subseteq F$, $f(X - F) \subseteq Y - W$. Since f is (γ, β^*)-pre-open, then H is β^*-pre-closed and $f^{-1}(H) = X - f^{-1}(f(X - F)) \subseteq X - (X - F) = F$.

(iii) $\Rightarrow$ (i). Let U be any γ-open set of X and $W = Y - f(U)$. Then $f^{-1}(W) = X - f^{-1}(f(U)) \subseteq X - U$ and $X - U$ is γ-closed. By (iii), there exists a β^*-pre-closed set H of Y containing W such that $f^{-1}(H) \subseteq X - U$. Then, $f^{-1}(H) \cap U = \emptyset$ and $H \cap f(U) = \emptyset$. Therefore, $Y - f(U) \supseteq H \supseteq W = Y - f(U)$ and $f(U)$ is β^*-pre-open in Y. This shows that f is (γ, β^*)-pre-open.

Corollary 4.2. Suppose $f: X \to Y$ is a (γ, β^*)-pre-open mapping and $\gamma: \tau \to P(X)$ is an open operation on τ. Then the following properties hold:

(i) $f^{-1}(cl_\beta(int_\beta(B))) \subseteq cl_\gamma(f^{-1}(B))$ for each set $B \subseteq Y$;
(ii) $f^{-1}(cl_\beta(V)) \subseteq cl_\gamma(f^{-1}(V))$ for each β-open set V of Y.

Proof. Follows from the Theorem 4.1(iii).

Theorem 4.3. Let f: $X \to Y$ be a mapping and γ: $\tau \to P(X)$ be an open operation on τ. Then the following conditions are equivalent:
(i) f is (γ, β^*)-pre-open;
(ii) $f(int_\gamma(A)) \subseteq pint_{\beta^*}(f(A))$ for $A \subseteq X$;
(iii) $int_\gamma(f^{-1}(B)) \subseteq f^{-1}(pint_{\beta^*}(B))$ for $B \subseteq Y$.

Proof. Straightforward from the Definition 4.1.

Theorem 4.4. Let f: $X \to Y$ be a bijective mapping. Then the following conditions are equivalent:
(i) f^{-1}: $Y \to X$ is (β^*, γ)-pre-continuous;
(ii) f is (γ, β^*)-pre-open;
(iii) f is (γ, β^*)-pre-closed.

Proof. Follows from the Definitions 3.1. and 4.1.

Theorem 4.5. Let f: $X \to Y$ be a mapping. Then the following statements are equivalent:
(i) f is (γ^*, β^*)-pre-open;
(ii) for each $x \in X$ and for every $A \in PO_{\gamma^*}(X)$ such that $x \in A$, there exists $B \in PO_{\beta^*}(Y)$ such that $f(x) \in$ B and $B \subseteq f(A)$;
(iii) for each $x \in X$ and for every γ^*-pre-neighborhood U of x in X, there exists a β^*-pre-neighborhood V of $f(x)$ in Y such that $V \subseteq f(U)$;
(iv) $f(pint_{\gamma^*}(A)) \subseteq pint_{\beta^*}(f(A))$, for all $A \subseteq X$;
(v) $pint_{\gamma^*}(f^{-1}(B)) \subseteq f^{-1}(pint_{\beta^*}(B))$, for all $B \subseteq Y$;
(vi) $f^{-1}(pcl_{\beta^*}(B)) \subseteq pcl_{\gamma^*}(f^{-1}(B))$, for all $B \subseteq Y$.

Proof. (i) $\Rightarrow$ (ii). Let A be a γ^*-pre-open set of x in X. Then $f(x) \in f(A)$. Since f is (γ^*, β^*)-pre-open, $f(A)$ is β^*-pre-open neighborhood of $f(x)$ in Y. Then by Definition 3.2(i), there exists $B \in PO_{\beta^*}(Y)$ such that $f(x) \in B \subseteq f(A)$.

(ii) $\Rightarrow$ (i). Let $A \in PO_{\gamma^*}(X)$ and $x \in A$. Then by assumption, there exists $B \in PO_{\beta^*}(Y)$ such that $f(x) \in B \subseteq f(A)$. Therefore $f(A)$ is a β^*-pre-neighborhood of $f(x)$ in Y and this implies that $f(A) = \cup_{f(x) \in f(A)} B$. Then by Theorem 2.1[10], $f(A)$ is β^*-pre-open in Y. Hence f is (γ^*, β^*)-pre-open.

(i) $\Rightarrow$ (iii). Let U be a γ^*-pre-neighborhood of $x \in X$. Then by Definition 3.2(i), there exists a γ^*-pre-open set W such that $x \in W \subseteq U$. This implies that $f(x) \in$

$f(W) \subseteq f(U)$. Since f is a (γ^*, β^*)-pre-open mapping, $f(W)$ is β^*-pre-open. Hence $V = f(W)$ is a β^*-pre-neighborhood of $f(x)$ and $V \subseteq f(U)$.

(iii) $\Rightarrow$ (i). Let $U \in PO_{\gamma^*}(X)$ and $x \in U$. Then U is a γ^*-pre-neighborhood of x. So by (iii), there exists a β^*-pre-neighborhood V of $f(x)$ such that $f(x) \in V \subseteq f(U)$. That is, $f(U)$ is a β^*-pre-neighborhood of $f(x)$. Thus $f(U)$ is a β^*-pre-neighborhood of each of its points. Therefore $f(U)$ is β^*-pre-open. Hence f is (γ^*, β^*)-pre-open.

(i) $\Rightarrow$ (iv). Let $x \in pint_{\gamma^*}(A)$. Then, there exists $U \in PO_{\gamma^*}(X)$ such that $x \in U \subseteq$ A. So $f(x) \in f(U) \subseteq f(A)$. Since f is (γ^*, β^*)-pre-open, therefore $f(U)$ is β^*-pre-open in Y. Hence $f(x) \in pint_{\beta^*}(f(A))$. Thus $f(pint_{\gamma^*}(A)) \subseteq pint_{\beta^*}(f(A))$.

(iv) $\Rightarrow$ (i). Let $U \in PO_{\gamma^*}(X)$. Then by (iv), $f(U) = f(pint_{\gamma^*}(A)) \subseteq pint_{\beta^*}(f(A)) \subseteq f(U)$ or $f(U) \subseteq pint_{\beta^*}(f(U)) \subseteq f(U)$. This implies that $f(U)$ is β^*-pre-open in Y. So f is (γ^*, β^*)-pre-open.

(iv) $\Rightarrow$ (v). Let B be any subset of Y. Clearly, $pint_{\gamma^*}(f^{-1}(B))$ is γ^*-pre-open in X. Also, $f(pint_{\gamma^*}(f^{-1}(B))) \subseteq f(f^{-1}(B)) \subseteq B$. Since f is (γ^*, β^*)-pre-open and by (iv), $f(pint_{\gamma^*}(f^{-1}(B))) \subseteq pint_{\beta^*}(B)$. Therefore $pint_{\gamma^*}(f^{-1}(B)) \subseteq f^{-1}(f(pint_{\gamma^*}(f^{-1}(B)))) \subseteq f^{-1}(pint_{\beta^*}(B))$. This gives $pint_{\gamma^*}(f^{-1}(B)) \subseteq f^{-1}(pint_{\beta^*}(B))$.

(v) $\Rightarrow$ (iv). Let $A \subseteq X$. By (v), it is found that $pint_{\gamma^*}(A) \subseteq pint_{\gamma^*}(f^{-1}(f(A))) \subseteq f^{-1}(pint_{\beta^*}(f(A)))$. This implies that $f(pint_{\gamma^*}(A)) \subseteq f(pint_{\gamma^*}(f^{-1}(f(A)))) \subseteq f(f^{-1}(pint_{\beta^*}(f(A)))) \subseteq pint_{\beta^*}(f(A))$. Consequently, $f(pint_{\gamma^*}(A)) \subseteq pint_{\beta^*}(f(A))$, for all $A \subseteq X$.

(v) $\Rightarrow$ (vi). Let B be any subset of Y. By (v), $pint_{\gamma^*}(f^{-1}(Y - B)) \subseteq f^{-1}(pint_{\beta^*}(Y - B))$. Then $pint_{\gamma^*}(X - f^{-1}(B)) \subseteq f^{-1}(pint_{\beta^*}(Y - B))$. As $pint_{\beta^*}(B) = Y - pcl_{\beta^*}(Y - B)$, therefore $X - pcl_{\gamma^*}(f^{-1}(B)) \subseteq f^{-1}(Y - pcl_{\beta^*}(B))$ or $X - pcl_{\gamma^*}(f^{-1}(B)) \subseteq X - f^{-1}(pcl_{\beta^*}(B))$. Hence $f^{-1}(pcl_{\beta^*}(B)) \subseteq pcl_{\gamma^*}(f^{-1}(B))$.

(vi) $\Rightarrow$ (v). Let $B \subseteq Y$. By (vi), $f^{-1}(pcl_{\beta^*}(Y - B)) \subseteq pcl_{\gamma^*}(f^{-1}(Y - B))$. Then, we have that $X - pcl_{\gamma^*}(f^{-1}(Y - B)) \subseteq X - f^{-1}(pcl_{\beta^*}(Y - B))$. Hence $X - pcl_{\gamma^*}(X - f^{-1}(B)) \subseteq f^{-1}(Y - pcl_{\beta^*}(Y - B))$. This gives $pint_{\gamma^*}(f^{-1}(B)) \subseteq f^{-1}(pint_{\beta^*}(B))$.

Theorem 4.6. Let $f: X \to Y$ and $g: Y \to Z$ be two mappings such that the composite mapping $g o f : X \to Z$ be a (γ^*, ρ^*)-pre-continuous.

(i) If g is (β^*, ρ^*)-pre-open injection, then f is (γ^*, β^*)-pre-continuous;

(ii) If f is (γ^*, β^*)-pre-open surjection, then g is (β^*, ρ^*)-pre-continuous.

Proof. Follows from the Definition 4.1.

Definition 4.2. A mapping $f: X \to Y$ is said to be (γ^*, β^*)-pre-homeomorphism, if f is bijective, (γ^*, β^*)-pre-continuous and f^{-1} is (β^*, γ^*)-pre-continuous.

Remark 4.2. From the Definitions 4.1 and 4.2, every bijective, (γ^*, β^*)-pre-continuous and (γ^*, β^*)-pre-closed map is (γ^*, β^*)-pre-homeomorphism.

Theorem 4.7. Let $f: X \to Y$ be (γ^*, β^*)-pre-homeomorphism. If X is γ^*-pre-$T_{\frac{1}{2}}$, then Y is β^*-pre-$T_{\frac{1}{2}}$.

Proof. Let $\{y\}$ be a singleton set of Y. Then, there exists a point x of X such that $y = f(x)$. It follows from the assumption and Theorem 5.5[10] that $\{x\}$ is γ^*-pre-open or γ^*-pre-closed. By Theorem 3.7(vi), $\{y\}$ is β^*-pre-open or β^*-pre-closed. This implies that Y is a β^*-pre-$T_{\frac{1}{2}}$ space.

Theorem 4.8. A mapping $f: X \to Y$ is (γ^*, β^*)-pre-closed if and only if $pcl_{\beta^*}(f(A)) \subseteq f(pcl_{\gamma^*}(A))$, for every subset A of X.

Proof. Suppose f is (γ^*, β^*)-pre-closed and let $A \subseteq X$. Since f is (γ^*, β^*)-pre-closed, therefore $f(pcl_{\gamma^*}(A))$ is β^*-pre-closed in Y. Since $f(A) \subseteq f(pcl_{\gamma^*}(A))$, therefore $pcl_{\beta^*}(f(A)) \subseteq f(pcl_{\gamma^*}(A))$. Conversely, suppose A is a γ^*-pre-closed set in X. By hypothesis, $f(A) \subseteq pcl_{\beta^*}(f(A)) \subseteq f(pcl_{\gamma^*}(A)) = f(A)$. Hence $f(A) = pcl_{\beta^*}(f(A))$. Thus $f(A)$ is β^*-pre-closed set in Y. This proves that f is (γ^*, β^*)-pre-closed.

Theorem 4.9. A mapping $f: X \to Y$ is (γ^*, β^*)-pre-closed if and only if $cl_\beta(int_\beta(f(A))) \subseteq f(pcl_{\gamma^*}(A))$, for every subset A of X.

Proof. Suppose f is (γ^*, β^*)-pre-closed and let $A \subseteq X$. Then $f(pcl_{\gamma^*}(A))$ is β^*-pre-closed in Y. This implies that $cl_\beta(int_\beta(f(pcl_{\gamma^*}(A)))) \subseteq f(pcl_{\gamma^*}(A))$. Then $cl_\beta(int_\beta(f(A))) \subseteq cl_\beta(int_\beta(f(pcl_{\gamma^*}(A))))$ gives $cl_\beta(int_\beta(f(A))) \subseteq f(pcl_{\gamma^*}(A))$. Conversely, suppose that A is a γ^*-pre-closed set in X. Then by hypothesis, $cl_\beta(int_\beta(f(A))) \subseteq f(pcl_{\gamma^*}(A))$. Since A is γ^*-pre-closed, $f(pcl_{\gamma^*}(A)) = f(A)$. Therefore $cl_\beta(int_\beta(f(A))) \subseteq f(A)$. Hence $f(A)$ is β^*-pre-closed in Y. This implies that f is (γ^*, β^*)-pre-closed.

Theorem 4.10. A mapping $f: X \to Y$ is a (γ^*, β^*)-pre-closed if and only if for each subset B of Y and each γ^*-pre-open set A in X containing $f^{-1}(B)$, there exists a β^*-pre-open set C in Y containing B such that $f^{-1}(C) \subseteq A$.

Proof. Let $C = Y - f(X - A)$. Since $f^{-1}(B) \subseteq A$, $f(X - A) \subseteq Y - B$. Since f is (γ^*, β^*)-pre-closed, then C is β^*-pre-open and $f^{-1}(C)= X - f^{-1}(f(X - A)) \subseteq X - (X - A) = A$. Conversely, let U be any γ^*-pre-closed set of X and $B = Y - f(U)$. Then $f^{-1}(B) = X - f^{-1}(f(U)) \subseteq X - U$ and $X - U$ is γ^*-pre-open. By the hypothesis, there exists a β^*-pre-open set C of Y containing B such that $f^{-1}(C) \subseteq$

$X - U$. Then $f^{-1}(C) \cap U = \emptyset$ and $C \cap f(U) = \emptyset$. Therefore, $Y - f(U) \supseteq C \supseteq B = Y - f(U)$ and $f(U)$ is β^*-pre-closed in Y. This shows that f is (γ^*, β^*)-pre-closed.

Theorem 4.11. Let $f: X \to Y$ be a bijective mapping. Then the following conditions are equivalent:

(i) f is (γ^*, β^*)-pre-closed;
(ii) f is (γ^*, β^*)-pre-open;
(iii) f^{-1} is (β^*, γ^*)-pre-continuous.

Proof. Follows from the Definition 4.1 and Theorem 4.5(vi).

Definition 4.3. Let $id: \tau \to P(X)$ be the identity operation. A mapping $f: X \to Y$ is said to be (id, β^*)-pre-closed if for any pre-closed set F of X, $f(F)$ is β^*-pre-closed in Y.

Theorem 4.12. If f is bijective mapping and $f^{-1}: Y \to X$ is (id, β^*)-pre-continuous, then f is (id, β^*)-pre-closed.

Proof. Follows from the Definitions 3.1, 4.1 and 4.3.

Theorem 4.13. Let $f: X \to Y$ be (γ^*, β^*)-pre-continuous and (γ^*, β^*)-pre-closed. Then

(i) for every γ^*-pg.closed set A of X, the image $f(A)$ is β^*-pg.closed;
(ii) for every β^*-pg.closed set B of Y , the set $f^{-1}(B)$ is γ^*-pg.closed.

Proof. Follows from the Theorems 2.1[10], 2.9(iii)[10], 3.7(v) and (vi).

Theorem 4.14. Let $f: X \to Y$ be (γ^*, β^*)-pre-continuous and (γ^*, β^*)-pre-closed.

(i) If f is injective and Y is β^*-pre-$T_{\frac{1}{2}}$, then X is γ^*-pre-$T_{\frac{1}{2}}$;
(ii) If f is surjective and X is γ^*-pre-$T_{\frac{1}{2}}$, then Y is β^*-pre-$T_{\frac{1}{2}}$.

Proof. Follows from the Theorem 4.13(i) and (ii).

Theorem 4.15. Suppose γ is a regular operation on τ. Then X is a γ^*-pre-$T_{\frac{1}{2}}$ space.

Proof. By Proposition 2.9[7], we have that (X, τ_γ) is a topological space. Now to prove X is γ^*-pre-$T_{\frac{1}{2}}$, it is enough to show that $\{x\}$ is γ^*-pre-open or γ^*-pre-closed.

Case (i): Suppose $\{x\} \in \tau_\gamma$. Then by Theorem 2.2[10], $\{x\}$ is γ^*-pre-open.

Case(ii): Suppose $\{x\} \notin \tau_\gamma$. Then $cl_\gamma(int_\gamma(\{x\})) = cl_\gamma(\emptyset) = \emptyset \subseteq \{x\}$. Hence $\{x\}$ is γ^*-pre-closed.

Theorem 4.16. Let X be a γ-regular space and $\gamma: \tau \to P(X)$ be a regular operation on τ. Then X is γ^*-pre-$T_{\frac{1}{2}}$ if and only if (X, τ_γ) is pre-$T_{\frac{1}{2}}$.

Proof. By Proposition 2.9[7], we have that (X, τ_γ) is a topological space. By Theorem 2.27[4], it is a pre-$T_{\frac{1}{2}}$ space. Conversely, if (X, τ_γ) is pre-$T_{\frac{1}{2}}$, then $\{x\}$ is pre-open or pre-closed in X. Hence it is γ^*-pre-open or γ^*-pre-closed in X and by Theorem 5.5[10], we have that X is a γ^*-pre-$T_{\frac{1}{2}}$ space.

Theorem 4.17. Let X be a γ-regular space and $\gamma: \tau \to P(X)$ be a regular operation on τ. Then X is γ^*-pre-T_2 if and only if X is pre-T_2.

Proof. Follows from the Theorem 2.3[10].

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