IDEAS FOR ADVANCING MATHEMATICALLY PRECOCIOUS YOUTH: A REVIEW OF THE LITERATURE

Brian Pablo Tello

*Eastern New Mexico University, **USA***

E-mail: brian.tello@enmu.edu

ABSTRACT

Mathematically Precocious Youth (MPY) are taught in a variety of ways in American educational system. This literature review paper explored which educational options may be best for MPY and the nation and reviewed many of the educational options available for MPY. The goal was to find which educational options provided MPY the best chance to be successful in school and beyond. The paper found many MPY were not being taught at the correct level and were not provided enough challenging material. MPY found great educational success as well as career success from acceleration programs, computer-based and distant learning courses, and mathematical competitions. The paper concluded acceleration set at the right level and pace was the best option to help MPY find educational success.

Keywords: *mathematically precocious youth, acceleration, distance-learning, mathematical competitions, grouping*

INTRODUCTION

The American educational system consists of students of extreme mixed abilities. All students desevere a chance to be educated to the maximum of their ability or capability. However, Beisser (2008) found there was a heavy focus on the lowest achieving students, and many gifted students and programs did not get equal attention or funding. In a report on the state of gifted education in America, Colangelo, Assouline, and Gross (2004) found the educational system was not doing enough to properly educate gifted students. An education that focuses on the specific needs of those students is a right, and gifted student should not be ignored.

Making sure that gifted students get the attention and education they deserve is not just beneficial for them, but also for the nation as a whole. Kell, Lubinski, and Benbow (2013, p. 458) found highly mathematically gifted students have "extraordinary potential for enriching society by contributing creative products and competing in global economics", and they believe the gifted can become trusted leaders. They also found that many gifted students went on to make great educational achievements: attending prestigious universities, creating many published materials, and earning higher than average amounts of PhDs. Many of those degree were in the technological and STEM fields, which is the direction the world is heading.

The world is becoming more technologically advanced every day. Math and science will power the future economy and America needs to foster its mathematically talented students (Broody, 2005). Bulgar (2008, p. 150), reiterates this point by emphasizing educators must continue to “prepare the most capable members of society” (gifted students) and enhance their abilities for good of the nation. As the world moves forward into the information age, America should pay attention to those individuals who have strong mathematical skills (Lubinski, Webb, Morelock, & Benbow, 2001). Therefore, this paper will look at Mathematically Precocious Youths (MYP). MPY are gifted students who have exceptional talents in the field of mathematics. Since the world is becoming more technologically advanced and the educational system is not meeting the need of gifted students, the focus of this paper will be on what can be done to better help MPY reach their true educational potential.

OPINIONS OF STUDENTS AND TEACHERS

The education of MPY is an important task. The assumption that MYP, like other gifted students, can make it on their own and need no extra focus is inaccurate. Diezmann and Watters (2002) stated gifted students, like all other students, need support and be provided challenging tasks. Gifted students have been shown to get bored and often underachieve when unchallenged in the traditional classroom (Colangelo, Assouline, & Gross, 2004). In a study on the student’s own opinions of the education they were receiving, Hallam and Ireson (2007) found that students did not believe there were being taught at the correct level, and desired more challenging work. These students also felt they were being correctly challenged in math and should be in more difficult classes. Students have also stated they became bored in an underchallenging environment (Preckel, Götz, & Frenzel, 2010). Gifted students need the correct level of challenge to help avoid boredom, so they can meet their full potential.

Teachers also need to pay attention to MPY. Leikin (2010) found teachers need to be more attentive, flexible, and reflective to the special needs of gifted students. Similar to the opinions of the students, Ayebo (2016, p. 23) found that teachers also believed gifted students needed to be challenged and needed rich educational material “that can arouse their natural curiosity”. Teachers and students agree more can be done to educate the gifted. However, the question remains as to what is actually being done to provide for MPY? The remaining portions of this paper look at several programs or ideas for advancing MPY.

IDEAS FOR HELPING MATHEMATICALLY PRECOCIOUS YOUTH SUCCEED

This paper attempts to review tools, programs, or methods used to help enrich and advance MPY by schools and educators. While doing research for this paper, it was difficult to find peer-reviewed literature covering specific curriculum, methods, or tools used to help MPY succeed. Nevertheless, there was a great deal of information on general concepts that have proved successful in maximizing the talents of MPY. From the literature reviewed, four ideas were shown to have promise and actual success at helping MPY find great success: computer programs and distance learning, acceleration, mathematic competitions, and select grouping. These programs or methods contained many similar themes which contributed to the MPY success. Those ideas will also be addressed below.

ACCELERATION

The first option looked at for advancing MPY is acceleration. "Acceleration is an intervention that moves students through an educational program at faster rates, or at younger ages, than typical", and can involve grade skipping, moving ahead to a higher subject level, entering a higher-level school early, or Advanced Placement courses (Colangelo, Assouline, & Gross, 2004, p. xi). A study by Therlfall and Hargreaves (2008) found that MPY can have problem solving skills four grades levels higher than their actual current grade. Some MPY who stay in a regular class find the class proceeds to slow, which can lead to boredom (Stanley, 1991). Stanley (1991) stated MPY need acceleration to satisfy their mental needs. In a case to support the need for acceleration, Sowell (1993) found MPY could learn mathematics more quickly than the typical student regardless of the means of instruction. Below are several successful examples of acceleration.

Acceleration can occur at all grade levels. MPY in high school can find success in college level acceleration programs. Dai and Steenbergen-Hu (2015) reviewed an early college acceleration program in China called Special Class for the Gifted Young (SCGY), which spanned over 34 years. The program is based on placing MPY into college early due to their exceptional talents. Dai and Steenbergen-Hu (2015) studied the program and found the program to be very successful for both individuals and society. The authors showed a majority of the participants had a positive experience in the program. The participants mentioned they enjoyed the flexibility of program because they were able to take the courses they wanted to. Dai and Steenbergen-Hu (2015) found the accelerated program was adaptive to individual differences. The authors showed many of the participants had great success at both education and careers. Many of the participants gained masters or doctoral degree and went on to careers in the STEMs fields. The authors stated for MPY who had self-direction, "acceleration is a natural option for them" (p. 16). Dai and Steenbergen-Hu (2015) concluded the MPY in the SCGY benefited from the early entrance into college, which lead the MPY to enrollment in prominent universities and to getting an early start on their careers.

The Study of Mathematically Precocious Youth (SMPY) is an ongoing longitudinal study that follows MPY through an acceleration process. The study uses acceleration to advance MPY and watches the outcomes, monitors the acceleration methods, and supports to the acceleration process (Swiatek, 2002). Stanley (1991) found many MPY found success in the SMPY. Stanley detailed a case study success of a 13-year-old who was a MPY and given acceleration courses early. The child went on to earn a PhD at 24 years of age. He stated many SMPY participants do excellent in National mathematics competitions, which can be used to measure the talents of gifted students to other students. Stanley continued to mention SMPY participants obtained high educational achievements.

In a more detailed look into the success of acceleration in the SMPY, Kolitch and Brody (1992) found many benefits of acceleration, and acceleration a valid choice for MPY. The authors showed how high school MPY participants succeeded at completing calculus courses ahead of schedule through acceleration. Kolitch and Brody (1992, p. 84) stated, "highly talented students can do well in mathematics courses taken several years earlier than is typical." The authors stated early acceleration could lead to more opportunities for even higher acceleration later. Kolitch and Brody (1992) suggested early acceleration could lead to early admittance into college. The authors mentioned many of the local schools the SMPY students attended cooperated in the acceleration process, which was helpful to the participants. After conducting the study, the authors discovered the SMPY students wanted to create their own challenging opportunities, MPY need curriculum that matched their needs, and MPY could benefit from additional work outside of school, such as clubs or competitions. In a follow-up study done ten years later, Lubinski, Webb, Morelock, and Benbow (2001) found that 93 percent of SMPY participants received a bachelor's degree, many having already earned master's, and 12% having earned a PhD. "In their early 20s, they (participants of the follow-up from then SMPY study) are beginning to accumulate achievements that are marked for individuals at this stage of development," (Lubinski, Webb, Morelock, & Benbow, 2001, p. 725). The SMPY showed acceleration can lead MPY to great educational success.

Kolitch and Brody (1992) made some recommendations to both students and school for implementing acceleration successfully. The authors recommend school administrators and teachers understand MPY can be very successful if accelerated and if the schools where these MPY attend, were open to the idea. The authors suggested schools should find was to provide acceleration beyond the calculus level. Kolitch and Brody (1992) also suggested schools provide extracurricular mathematical activities for the MPY to participate in. In addition to, Kolitch and Brody (1992) have recommended accelerated students should take their acceleration courses in the proper sequence, and MPY should be aware they may run out of advanced courses to take. Kolitch and Brody (1992) cautioned students should be wary about taking accelerated courses in high school if there will be a conflict with courses taken at the colligate level, such as retaking courses. Also, the authors mentioned participation in extracurricular mathematical activities can be beneficial.

To look at the outcomes of acceleration in the SMPY, Swiatek (2002) conducted a 10-year longitudinal study of the SMPY program. She found, with acceleration, younger students had the same amount of educational achievement as older average-ability students. Swiatek (p.2) also showed students who took accelerated courses "did not suffer academically" and gained "speed in their educational preparation". Her research discovered students who choose acceleration did not have educational gaps in those accelerated subjects. In support of previously mentioned findings, Swiatek showed acceleration lead to higher than average completion of college and high attendance in graduate school. She mentioned students who took accelerated courses maintained their interest in the subject matter (mathematics) and continued with their educational pursuits (Swiatek, 2002).

MPY need educational opportunities to match their needs. Acceleration can be one way to achieve that task. Acceleration allows MPY to take classes at the pace that best suits their capability. The high levels of educational and occupational success attained by MPY show acceleration can produce great results. Colangelo, Assouline, and Gross (2004) stated acceleration was the best intervention for gifted students. MPY in the SMPY acceleration program have become educators at the colligate level, taken leadership jobs, and published educational research, all of which is beneficial to society as a whole. Acceleration can work to enrich the academic careers of MPY if done correctly.

COMPUTER PROGRAMS AND DISTANCE LEARNING

Acceleration has been shown to be effective in progressing MPY. Computer programs or distance learning can provide a means for acceleration. Olszeski-Kubilius (2010) stated distance learning could be advantageous for gifted students because it can provide advanced courses which can be more individualized. For the purpose of this paper, computer programs and distance learning will refer to material presented in a non-traditional way (traditional referring to direct instruction given by a teacher in a typical classroom) either solely through the computer (on-line courses), or a majority of the program or course being computer based, with additional supplementary materials, such as DVD videos and associated textbooks. The computer programs or distance learning can take place at multiple locations such as student's primary school or home. Wallace (2009) found that distance learning could be a good option for those who are home schooled or those who will participate independently.

Distance learning can be used to enrich, supplement, or accelerate the student's current curriculum. Gifted students who finish regular classwork quickly may be given extra busy work, which may do little to enrich their education (Basister & Kawai, 2018). Using distance learning or computer programs can be a way to fill that time with enriching material. In a study to determine the effectiveness of mathematical distance learning on gifted elementary students, Suppes, Holland, Hu, and Vu (2013) found allocating 20 minutes of distance learning a day yielded positive educational gains. However, it must be noted those gains were only significant for the highest performing students. Rotigel and Fello (2004) stated in-class computed programs provide a way for gifted students to gain enrichment. Dimitriadis (2012) found gifted teachers can use computer-based programs at the beginning of class to provide more challenging work to their students. Deal and Wismer (2010), found on-line programs can be used to match the students' interest or needs, and used to connect students to advanced classes that are great distances away. Below is a review of several successful computer based or distance learning programs used to enrich MPY.

EDUCATIONAL PROGRAMS FOR GIFTED YOUTHS

The Education Program for Gifted Youth (EPGY) from Stanford University found success in advancing gifted students through computer programs. Ravaglia, Suppes, Stillinger, and Alper (1995) found EPGY helped MPY advance at their own pace. The EPGY program was an interactive course with lessons, texts, questions, lectures, and assessments. Participants in the EPGY had to find ways to manage their educational habits more than they would in normal classroom and thus go at their own pace (Ravaglia, Suppes, Stillinger, and Alper, 1995). The authors found both boys and girls had equal success taking the computer program, and both sexes were able to complete advanced math courses. Many students were able to pass multiple advanced classes, which might not have been typically available at their traditional school. The authors mentioned students could work at home alone, and advanced students could work independently instead attending a college class with much older students. The EPGY program also allowed students to start at any time, including the summer.

Ravaglia, Suppes, Stillinger, and Alper (1995) found the EPGY computer program benefited schools and students by allowing schools to offer more advanced mathematical courses and allowed students to take additional mathematical courses beyond what was normally available. In addition to, the EPGY did not have a great impact on school funds. Ravaglia, Suppes, Stillinger, and Apler (1995) stated the program allowed gifted students to work at their own pace and take the necessary classes as to not waste time taking unneeded classes in the future.

Another study focused on MPY development with taking distance learning classes and the participants' opinions of the program. Olszewski-Kubilius and Lee (2004) studied mathematically gifted students in grade six through 12 who took advanced distance learning. The authors found distance learning participants received higher than the national average scores on Advanced Placement exams and completed the courses in three to six months. In addition to, nearly half the students received high school credit for the distance learning classes they took. It must be mentioned, Olszewski-Kubilius and Lee (2004) emphasized many students did not ask the schools if they could receive credit for the classes they passed. The authors found the students were overall satisfied with the program and mentioned they took the classes to enrich themselves, to be able to go their own pace, or because the courses were not offered at their school. The students also stated the courses were at the correct level of challenge (Olszewski-Kubilius & Lee, 2004). The participants found the courses to be more demanding and rewarding than regular coursework.

A study by Wallace (2009) sought to find the effectiveness of distance learning programs on students of different ages, the reason students took the class, and participants' opinions of the courses. Wallace (2009) stated distance learning was a good way for gifted students to take courses not available at the student's school. The author found MPY had success in taking the courses regardless of what grade they were in. The students were found to have positive views of the courses, and believed the course were at an appropriate level of challenge. In addition to, a majority of the students who took the distance learning courses indicated they "became more interested in the material" (Wallace, 2009, p. 308). Wallace (2009) found distance learning "can be an effective approach to accelerate or enrich the academic opportunities available to gifted students in grades K through 12" (p. 312). The author stated distance learning could be used to individualize challenge levels, a good option at schools that do not provide higher level classes and be used at all grade levels.

Distance learning and computer programs, much like acceleration, can provide a way for MPY to take the courses that best fit their mathematical talents. MPY can benefit from distance learning by being able to take courses that match their interest and at a pace that best fits their skills. Distance learning could be a good option for schools who may not be otherwise able to provide quality instruction or curriculum to MPY. In addition to, distance learning can be provided by schools at little cost. MPY can use distance learning to get ahead and find challenging mathematical work outside of the typical class. Furthermore, MPY may not need or want to take the basic classes offered at their school because of their mathematical talent, and distance learning can provide challenging courses to continue their educational enrichment.

COMPETITIONS

Providing MPY opportunities to work on their higher-level mathematical skills is a must. Many schools do not have or cannot provide the higher, enriching classes MPY need. In addition to, even those schools that can provide acceleration or distance learning courses, should do more. Schools should offer extracurricular activities for interested students (Kolitch and Brody, 1992). A proven way to add enriching mathematics for MPY is through mathematical competitions. These mathematical competitions provide enrichment in another form for MPY.

Competitions can be used to supplement the education of MPY. Competitions provide an opportunity to improve independent learning as well as build skills working on a team, while building problem solving skills (Bicknell, 2008). Bicknell studied a group of students over a two-year period looking at the educational benefits to gifted students and opinions of students, teachers, and parents from mathematical competitions. He found all participating teachers valued the competitions as a part of the mathematics curriculum. Furthermore, the study showed MPY liked the competitions because it allowed them to gauge their mathematical skills compared to other gifted students in and out of the country. Parents, students, and teacher all had favorable opinions of the team competitions. However, Bicknell warned that several teachers stated competitions are not for everyone because some students would not feel comfortable due to the naturally competitive nature of competitions. Bicknell stated competitions bring together gifted students who can build friendships and gain encouragement. He concluded competitions can play a large role in creating excitement and enrich the mathematics curriculum for gifted students (Bicknell, 2008).

Mathematical competitions can enrich the curriculum of MPY, but they can do much more. Competition are a way to recognize the achievements and celebrate the accomplishments of MPY (Olszewski-Kubilius, 2010; Bicknell, 2008). Competitions give students “the extra incentive to work hard and consequently to improve their performance,” (Udvari & Schneider, 2000, p. 213). Competitions can fit a student’s specific interest, allow gifted students to work with other gifted students, foster increased mathematical performance, give MPY creative outlets, and allow gifted students to work with mentors and talented professionals (Olszewski-Kubilius, 2010). One competition (The Knowledge Master Open) allows students to compete using a computer program right in their own class (Riley & Karnes, 1998). In addition to, competitions allow MPY opportunities to expand their skills and knowledge in the mathematic fields (Riley & Karnes, 1998). There are numerous benefits of MPY competing in mathematical competitions. This paper examines a few instances in which schools and countries have used competitions to enrich the mathematic knowledge of their gifted students.

When competitions are implemented in a quality fashion, the impact can be a very positive for MPY. In a longitudinal study that examined the role of competitions in Hungary spanning 20 years, Stockton found competitions played a large role in mathematical education the that country. The competitions in Hungary promoted interest, engagement, and involvement in mathematics. She also found competitions provided needed enrichment, creativity, and problem-solving skills. The competitions used in Hungary are used to enrich the curriculum of MPY. The Hungary journal KöMaL provides a yearlong mathematical competition, which every month poses original and advanced questions and mathematical questions from other competitions. The KöMaL allows MPY to pursue and tackle complex mathematical questions during the entire school year. These types of competitions allow MPY to engage in mathematical pursuits while continually challenging them. The success of the KöMaL competition is one where “the tradition of excellence breeds excellence,” (p. 55). As stated earlier, many schools do not have highly advanced mathematics courses to offer MPY students who may have taken all the available courses. The competitions, like the ones in Hungary, provide MPY an outlet to pursue mathematical interest when no additional advanced classes are available (Stockton, 2012).

The competitions in Hungary do more than provide enrichment for the mathematically gifted. The numerous competitions can also be used as a tool to identify MPY (Stockton, 2012). Then after identification, the competitions can provide the enrichment needed to advance MPY. She stated the competitions are used to provided extra support and extracurricular activities, which are created to help students prepare for competitions. Competitions help engage MPY while developing their mathematical talent. There are competitions in Hungary available for all skill levels with the intent to make mathematics “fun, interesting, and accessible to the average student,” (p. 43). In addition to all these benefits, the questions from competitions (such as the questions provided by the KöMaL journal) can be used as lessons or material in the regular class. Hungary has found a way to enrich, engage, and stimulate MPY development through mathematical competitions (Stockton, 2012).

The goal of many mathematical competitions is to develop the talents and skills of MPY. However, what are the benefits of developing the mathematical skills

MPY? A study by Campbell and Walberg (2011) looked at the role competitions played on the future success of MPY who participated in those competitions. The competitions in the study were the annual Mathematics, Chemistry, and Physics Olympiads, and the competitors in the Olympiads will be referred to as Olympians. One goal of competition is in developing talent, and then after development, "the talent is expected to contribute to society," (Campbell & Walberg, 2011, p. 8). The authors stated many competitors, not just the winners, benefit from competitions because they learn to do in-depth research. The results of the study revealed many Olympians went on to continue their education at prestigious universities. These Olympians also received advanced degrees, with 525 of participants having "completed, or in the process of completing, doctorate degrees", (p. 12). They also found many Olympians used those degrees to become teachers or researchers at the colligate level, began work in technology (computer related) fields, became engineers, or began work in the private business sector. In addition to, the Olympians published large amounts of literature, and published at a much higher rate than average. As part of the study, the authors found when Olympians were asked if they would have "turned out as well without the Olympiad program", "both the Olympians (76%) and their parents (70%) expressed the view that they would not have accomplished as much without the program," (p. 14). They concluded many Olympians do fulfill their potential, and work in leadership position, which serves the national interest (Campbell and Walberg, 2011).

Competitions provide a way to enrich MPY. Campbell and Walberg (2011) argued competitions are needed because many schools cannot provide the curriculum or resources MPY need. There are many competitions beyond the Olympiads which schools can choose from. These competitions can build important skills used later in life and can enrich MPY development. The authors recommended schools use competitions to enrich MPY, and that females and minorities need a chance to compete. The authors also suggested teachers who produce winners should be recognized for their work (Campbell & Walberg, 2011).

Competitions provide an additional outlet for MPY to gain mathematical enrichment. Competitions give MPY access to challenging, advanced, and engaging mathematical materials. MPY who compete get the opportunity to work with other talented students, conduct research, and work with mentors. Competitions have shown to have positive effects on the continuing education of MPY. Many Olympians went on to gain advanced degrees, work in prestigious fields, and become published authors. Schools can use competitions to continue the advanced education MPY deserve. Competitions can be used to supplement the mathematical curriculum, provide ongoing enrichment to MPY, and be an engaging way to allow MPY to use their talents or find outlets for their interests.

A WORD ON GROUPING

Another idea that emerged from a review of the literature was grouping. For this paper, grouping refers to gifted students working together in a mixed-ability classroom, which is a classroom with students of all skill levels, or grouping done by a pull-out program, which typically happened once a week for several hours.

MPY learn differently than other students (McAllister & Plourde, 2008). Gifted students need the opportunity to work with other gifted students. If gifted students do not get the opportunities to work with other gifted students, they are missing out on learning through discourse and may feel isolated (Diezmann & Watters, 2002). Doing research on what is essential for MPY, McAllister and Plourde (2008, p. 46) concluded gifted students who can work with other gifted students "are able to discuss high-level mathematical thinking with intellectual peers who understand their way of thinking." Dimitriadis (2012) found that a pullout program in England for MPY allowed the students to have more access to differentiated learning and extra enrichment. Dimitriadis (2012) also found the students in the pullout program had positive attitudes toward the program and the lessons involved. Furthermore, MPY need the opportunity to work on more complex problems and need as much attention from the teacher as other students, which suggest grouping or pull programs may be needed (Dimitriadis, 2012).

Stanley (1991) stated that it should be noted MPY can get bored with basic material, a study by Preckel, Götz, and Frenzel (2010) on ability grouping and the effects of boredom, found gifted students grouped together in a mixed ability classroom did not have higher levels of boredom than the other students. Further, a study by Shayshon, Gal, Tesler, and Ko (2014) found although teachers of MPY agreed with grouping students, they did not want those gifted students to be in a separate class.

The main goal identified from the literature was grouping provided MPY the opportunity to work with other MPY. The gifted students benefit from having the chance to have discussions with others with the same mathematical talent. Grouping and pullouts programs can provide MPY with additional enriched curriculum beyond the normal classroom, as well as individualized instruction. However, teachers believed grouping should occur in the regular, mixed-ability classroom. After a review of the literature, acceleration looked like a better fit for MPY than grouping.

COMMON THEMES

The mentioned ideas above were shown to have a positive impact on the educational success of MPY. However, it is important to acknowledge that some common traits were continually mentioned while reviewing the literature on those successful programs. Those traits emphasized important hallmarks of the programs. The main traits of a program for helping MPY become successful were: they needed to be challenging, pacing was based on the individual, the program matched the needs or interests of the gifted student, counseling was provided, and or MPY had opportunities to work with other mathematically talented students.

Providing all students with the appropriate level of challenge should be a goal of all educators. A study by Hallam and Ireson (2007) found students wanted to be challenged when it came to their mathematics courses. MPY need to be challenged early or when the courses begin to get challenging on their own, MPY can lose self-esteem, quit taking risks, and can become underachievers (McAllister & Plourde, 2008). Preckel, Götz, and Frenzel (2010) found gifted students can get bored if they are underchallenged. Furthermore, there is need to challenge MPY to make sure they can achieve as much as possible. Leikin (2011, p. 180) stated, “mathematical challenge is a necessary condition for realization of mathematical potential,”. However, Ayebo (2016) pointed out the level of challenge must be appropriate; not to high or to low. Providing the appropriate level of challenge should be considered for all programs dealing with MPY.

Pacing was another issue that was continually mention during the review of literature. Students of different levels learn at different rates and it is the same for MPY. Johnson (2000) stated gifted students need individualized pacing because everyone learns differently. During the SMPY, Stanley (1991) found adjustments in individualized pacing was needed because many MPY were able to learn at an extremely fast educational pace whereas others needed more time. In addition to, students wanted the pace of curriculum to match their mastery level (Lubinski, Webb, Morelock, & Benbow, 2001). Allowing gifted students to learn at their own pace can be very beneficial. Students can make tremendous gains if they can move at their own pace (Ravaglia, Suppes, Stillinger, & Alper, 1995). MPY benefit when allowed to go at the pace that matches their skill level.

Much like pacing, matching programs, courses, or competitions with the interests or talents of gifted students was mentioned quite often during the review of literature. MPY benefit from moving at their own pace, but also from receiving mathematics enrichment that matches their needs and interests. Allowing MPY to take the classes based on their interests may reduce their frustration in school (Rotigel & Fello, 2004). Stockton (2012) recommended schools offer competitions to the curriculum because they promote student interest and specifically can motivate MPY interest in mathematics. Students themselves recommend taking courses that fit their interests. Former participants in SMPY “advised others (future SMPY participants) to pursue the mathematics education uniquely matched to their own needs and interests,” (Kolitch & Brody, 1992, p. 84). The needs and interest of MPY must be taken into consideration, and the mathematical programs should match those needs and interests.

Another key aspect of the programs was the opportunity for MPY to work with other MPY. There are benefits from having MPY work with their peers both socially and academically. Students in the SMPY said they enjoyed working with other mathematically talented students (Sowell, 1993). “Competitions can serve as a way of bringing students of ‘like minds’ together so that they find friendship, inspiration, and encouragement from working with others,” (Bicknell & Riley, 2012, p. 7). Sowell (1993) also found greater productivity for those MPY who worked in greater amounts with other MPY. Many of the ways mentioned to help enrich and advance MPY allowed, or even required, gifted students to work together. Distance learning provided forums by which MPY could work with their peers. (Olszewski-Kubilius & Lee, 2004). Riley (2012) found competitions provided forums that could be used for talented students to connect with other talented individuals and develop new understandings in their field of interest. Sowell (1993) suggested having gifted students work together played a part in the success the acceleration program. Diezmann and Watters (2002) argued gifted students need opportunities to work with peers to gain knowledge from the discourse with like minds. Allowing gifted students the opportunity to work with their peers is crucial for the educational development of MPY.

Counseling and mentoring was another item touched on for creating a successful program for advancing MPY. For the context of this paper, counseling and mentoring dealt both with giving educational guidance and support to MPY and providing emotional and social support. The Chinese program SCGY provided ongoing counseling and guidance for no charge to all the students enrolled in the program (Dai & Steenbergen-Hu, 2015). In addition to, Dai and Steenbergen-Hu (2015) found some of the participants in the SCGY said they felt lost at some point of school and wanted counseling and guidance. Brody (2005) found MPY sought counseling to help decide on what course to take, acceleration options, summer programs, or information entering college early, and how to interact with others. A focus of counseling for some MPY dealt with "dealt with ways to accelerate and enhance their educational programs," and encourage MPY to interact more with peers who share the same interests (Brody, 2005, p. 88). Counseling MPY should be geared toward guiding the students to meet their educational goals and addressing their social and emotional needs along the way.

DISCUSSION

Mathematically precocious youth need assistance in meeting their educational needs, just as any other student. The notion gifted students can succeed on their own is just not true. The nation benefits from the providing a sound yet challenging education to MPY. However, not enough is being done to assist MPY in their educational careers. Colangelo, Assouline, and Gross (2008) found the American educational system has failed to meet the educational needs of gifted students. This needs to change. The question should change from, "why should educators and schools pay extra attention to MPY" to "how can educators and schools provide the proper enriching education to MPY?".

Gifted students, and especially MPY, need special attention paid to their education for them to have the best chance of reaching their full potential. Colangelo, Assouline, and Gross (2008, p. 2) stated "educational equity does not mean educational sameness. Equity respects individual differences in readiness to learning and recognizes the value of each student." MPY are different from the average student and need to be taught according to their capabilities. Gifted students have amazing potential but only if they are given a chance to succeed.

The research stated above has shown there are numerous programs available that can enrich the education of MPY. It was stressed that providing MPY with challenges, finding courses to match their interests, allowing MPY to work with peers, and adjusting pacing for individuals, was key to assisting MPY. Grouping gives MPY a chance to work with other MPY. However, the success of grouping can depend on the quality of work given to those MPY, and the limited amount of time in grouping through pull-out or in regular classes may have an effect quality of learning. Competitions are a way for MPY to participate in extracurricular activities that match their interests. These mathematical competitions can also provide enriching challenges for MPY, which are sometimes not found in a typical class. In addition to, competitions can provide supplemental material used in lessons, that can be used as enrichment material for MPY who finish early instead of just giving them busy work and provide ways for MPY to work with their peers or even mentors. Acceleration may be the best option for MPY. Acceleration allows MPY to take the challenging courses they need or enjoy, and to do so at their pace. Furthermore, acceleration can easily be done in a willing school or provided through distance learning at any available computer with internet access. Acceleration can give the MPY a chance to get a head start on college and even a promising career.

CONCLUSIONS

Mathematically precocious youth need assistance to find educational success. It is recommended that schools provide as many options as possible to enrich the education of MPY. Providing acceleration can be easy cost-effective way to do just that. Colangelo, Assouline, and Gross (2008, p. 2) stated "acceleration is a virtually cost-free intervention." Acceleration may save the students money as well, because they will not have to pay thousands of dollars to take the same classes in college (Colangelo, Assouline, & Gross, 2008). Acceleration for MPY should happen as soon as possible. MPY should not have to find boredom in classes which provide them with little challenge or educational value. The SMPY, SCGY, and EPGY all used some form of acceleration or distance learning and found the MPY participants in those studies had great success. Acceleration and distance learning should be options for all MPY regardless of what school they attended.

To go along with acceleration, providing MPY the opportunities to participate in mathematical competitions should also be an option. Competitions are a great way to allow MPY to enjoyably pursue their interest. Competitions provide excellent challenges, which can give much needed enrichment to MPY. Also, because there are many mathematical competitions, schools should be able find one that fits their educational goals. Competitions also present MPY with a key element mentioned throughout this paper; the ability to work with other MPY. In all, when MPY succeed, whether from acceleration or mathematical competitions, the nation will succeed.

REFERENCES

1) Ayebo, A., (2016). Teachers' Perceptions on Identifying and Catering to the Needs of Mathematically Gifted and Talented Students. *Journal Pendidikan Malaysia,* 41(1), 19-24.
2) Basister, M. P. & Kawai, N., (2018). Japan's educational practices for mathematically gifted students. *International Journal of Inclusive Education,* 1, 1-29.
3) Beisser, S. R., (2008). Unintended consequences of No Child Left Behind mandates on gifted students. *Forum on Public Policy,* n2, 1-13.
4) Bicknell, B., (2008). Gifted students and the role of mathematics competitions. *Australian Primary Mathematics Classroom,* 13(4) 16-20.

5) Bicknell, B. & Riley, T., (2012). The role of competitions in a mathematical programme. *New Zealand Journal of Gifted Education,* 17(1), 25-34.
6) Brody, L. E., (2005). The study of exceptional talent. *High Abilities Study,* 16 (1), 87-96.
7) Bulgar, S., (2008). Enabling more students to achieve mathematical success: A case study of Sarah. *Creativity, Giftedness, and Talent Development in Mathematics,* which volume, 133-154.
8) Campbell, J. R. & Walberg, H. J., (2011). Olympiad studies: Competitions provide alternatives to developing talents that serve national interests. *Roeper Report,* 33, 8-17.
9) Colangelo, N., Assouline, S. G., & Gross, M. U. M. (Eds.), 2004. *A nation deceived. How schools hold back America's brightest students,* (Vol. 2), Iowa City; University of Iowa.
10) Dai, D. Y. & Steenbergen-Hu, S., (2015). Special class for the gifted young: A 34-year experiment with early college entrance programs in China. *Roeper Review,* (37), 9-18.
11) Deal, L. J. & Wismer, M. G., (2010). NCTM principles and standards for mathematically talented students. *Gifted Child Today,* (33), 55-65.
12) Diezmann, C. M. & Watters, J. J., (2002). Summing up the education of mathematically gifted students. In B. Barton, K.C. Irwin, M. Pfannkuch, and M.J. Thomas (eds). *Mathematics Education in the South Pacific*. Proceedings of the 25th Annual conference of the Mathematics Education Research Group of Australasia, Auckland, (pp.219-226). Sydney: MERGA.
13) Dimitriadis, C., (2012). How are schools in England addressing the needs of mathematically gifted children in primary classrooms? A review of practice. *Gifted Child Quarterly,* 56(2), 59-76.
14) Hallam, S. & Ireson, J., (2007). Secondary school pupils' satisfaction with their ability grouping placements. *British Educational Research Journal,* 33(1), 27-45.
15) Johnson, D. T., (2000). Teaching mathematics to gifted students in a mixed-ability classroom. (Report No. EDO-EC-00-3). Reston, VA: Eric Clearinghouse on Disabilities and Gifted Education. (ERIC Document Reproduction Service No. ED441302)
16) Kell, H. J., Lubinski, D., & Benbow, C. P., (2013). Who rises to the top? Early indicators. *Psychological Science,* 24(5), 648-659.
17) Kolitch, E. R. & Brody, L. E., (1992). Mathematics acceleration of highly talented students: An evaluation. *Gifted Child Quarterly,* 36(2), 78-86.
18) Leikin, R., (2011). The education of mathematically gifted students: Some complexities and questions. *The Montana Mathematics Enthusiast,* 8(1-2), 167-188.
19) Leikin, R., (2010). Teaching the mathematically gifted. *Gifted Education International,* 27, 161-175.

20) Lubinski, D., Webb, R. M., Morelock, M. J., & Benbow, C. P., (2001). Top 1 in 10,000: A 10-year follow-up of the profoundly gifted. *Journal of Applied Psychology,* 86(4), 718-729.
21) McAllister, B. A. & Plourde, L. A., (2008). Enrichment curriculum: Essential for mathematically gifted students. *Education,* 1, 40-49.
22) Olszewski-Kubilius, P. & Lee, S. Y., (2004). Gifted adolescents' talent development through distance learning. *Journal for the education of the Gifted,* 28(1), 7-35.
23) Olszewski-Kubilius, P., (2010). Special schools and other options for gifted STEM students. *Roeper Review,* 32, 61-70.
24) Preckel, F., Götz, T., & Frenzel, A., (2010). Ability grouping of gifted students: Effects on academic self-concept and boredom. *British Journal of Educational Psychology,* 80, 451-472.
25) Ravaglia, R., Suppes, P., Stillinger, C., & Alper, T., (1995). Computer-based mathematics and physics for gifted student. *Gifted Child Quarterly,* 39(1), 7-13.
26) Riley, T. L. & Karnes, F. A., (1998). Mathematics + competitions = a winning formula. *Gifted Child Today,* 21(4), 42-46.
27) Rotigel, J. V. & Fello, S., (2004). Mathematically gifted students: How can we meet their needs? *Gifted Child Today,* 27(4), 46-51, 65.
28) Shayshon, B., Gal, H., Tesler, B., & Ko, E., (2014). Teaching mathematically talented students: A cross-cultural study about their teachers' views. *Educational Study of Mathematics,* 87, 409-438.
29) Sowell, E. J., (1993). Programs for mathematically gifted students: A review of empirical research. *Gifted Child Quarterly,* 37(3), 124-131.
30) Stanley, J. C., (1991). An academic model for educating the mathematically talented. *Gifted Child Quarterly,* 35(1), 36-46.
31) Stockton, J. C., (2012). Mathematical competitions in Hungary: Promoting a tradition of excellence & creativity. *The Mathematics Enthusiast,* 9(1-2), 37-58.
32) Suppes, P., Holland, P. W., Hu, Y., & Vu, M., (2013). Effectiveness of an individualized computer-driven online math K-5 course in eight California Title 1 elementary schools. *Educational Assessment,* 18, 162-181.
33) Swiatek, M. A., (2002) A decade of longitudinal research on academic acceleration through the Study of Mathematically Precocious Youth. *Roeper Review,* 24(3), 141-147.
34) Threlfal, J. & Hargreaves, M., (2008). The problem-solving methods of mathematically gifted and older average-attaining students. *High Ability Students,* 19(1), 83-98.
35) Udvari, S. J. & Schneider, B. H., (2000). Competition and the adjustment of gifted children: A matter of motivation. *Roeper Review,* 22(4), 212-216.
36) Wallace, P., (2009). Distance learning for gifted students: Outcomes for elementary, middle, and high school aged students. *Journal for the Education of the Gifted,* 32(3), 295-320.